# A Creep Equation Based on Thermal Activation

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**Abstract:** The creep equation proposed in so called  $\theta$  projection concept is developed in the concept of thermal activation of creep. The measured creep curves of A286 alloy are fitted by the equation. The activation energy in the equation is evaluated, and creep rupture lives are predicted. The results are confirmed by creep tests of up to 13 years.

Key words: creep; creep curve equation; creep rupture life; A286 superalloy

It has been realized that creep curve determination experiment can be used to make prediction of long term creep properties of materials. This prediction is always based on the extrapolation of a creep equation determined by some short term tests. Evans and Wilshire<sup>[1,2]</sup> have introduced the following creep equation

 $\varepsilon = \varepsilon_0 + \theta_1 \{1 - \exp(-\theta_2 t)\} + \theta_3 \{\exp(\theta_4 t) - 1\}$  (1) where  $\theta_i$  (i = 1, 2, 3, 4) are determined by temperature and stress as following

$$\ln \theta = a + b T + c \sigma + d \sigma T \tag{2}$$

This is called  $\theta$  projection concept. The advantage of this method has been shown to accurately describe the creep curves and predict creep rupture life of more than 10 years based on the creep tests of less than 3 months<sup>[3]</sup>. K Maruyama, et al. <sup>[4,5]</sup> modified Eq.(1) as

 $\varepsilon = \varepsilon_0 + A\{1 - \exp(-\alpha t)\} + B\{\exp(\alpha t) - 1\}$  (3) They also proposed<sup>[5]</sup> that in Eq. (3) was related to creep activation energy.

In the present paper, Eq. (1) is to be developed in the concept of thermal activation of creep, so that the relation between  $\alpha$  in Eq. (3) and activation energy can be well explained. Creep curve determination experiment is made for A286 alloy to evaluate the activation energy of creep and predict creep rupture life of the material. The results will be examined by creep date sheet published by NRIM of Japan<sup>[6]</sup>.

## 1 Derivation of Creep Equation

There are two mechanisms in the creep of a material, one being creep hardening and another being creep weakening<sup>[2]</sup>. In terms of microstructure, creep harden-

ing is related to the exhaustion of movable defects such as dislocations, while creep weakening is due to the accumulation of damage such as cavities. Strains produced by these two mechanisms are discussed respectively.

In creep hardening, it is supposed that there are N dislocations at time t in unit volume of material, then the rate of activation of dislocation is

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \alpha_1 \tag{4}$$

Integration of Eq.(4) gives

$$N = N_0 \exp(-\alpha_1 t) \tag{5}$$

where  $N_0$  is the number of movable dislocations at the beginning of creep, and  $\alpha_1$  is the ratio of activation of dislocation. According to the theory of thermal activation,  $\alpha_1$  can be written as

$$\alpha_1 = f(\sigma) \exp(-Q_1 / RT) \tag{6}$$

where  $Q_1$  being the activation energy of dislocation,  $f(\sigma)$  being the function of stress  $\sigma$ . At time t, the strain produced by the movement of dislocations is proportional to the number of dislocations exhausted, or  $(N_0 - N)$ , so that

$$\varepsilon_1 = A\{1 - \exp(-\alpha_1 t)\} \tag{7}$$

where A is coefficient, proportional to  $N_0$ . As  $N_0$  is related to the creep condition, A is the function of  $\sigma$  and T.

For the mechanism of creep weakening, it is proposed that at time t the damage is  $\Phi$ , then the rate of damage increase is proportional to  $\Phi$ 

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \alpha_2 \Phi \tag{8}$$

Integration of Eq.(8) gives

$$\boldsymbol{\Phi} = \boldsymbol{\Phi}_0 \exp(\alpha, t) \tag{9}$$

where  $\Phi_0$  is the damage of the material at the begin-ning of creep. In fact, the increase of damage is also depend on thermal activation, so that  $\alpha_2$  can be written as

$$\alpha_{\gamma} = f(\sigma) \exp(-Q_{\gamma} / RT)$$
 (10)

It is reasonable to resume that the strain produced by the damage is proportional to the damage accumulated in creep, or  $(\Phi - \Phi_0)$ 

$$\varepsilon_{1} = B\{\exp(\alpha_{1}t) - 1\}$$
 (11)

where *B* is coefficient, proportional to  $\Phi_0$ . The total strain pro-duced by creep is the sum of  $\varepsilon_1$  and  $\varepsilon_2$ .

$$\varepsilon = \varepsilon_0 + A\{1 - \exp(\alpha_1 t)\} + B\{\exp(\alpha_2 t) - 1\}$$
 (12) where  $\varepsilon_0$  is the instantaneous plastic flow occurred at the beginning of creep.

Eq. (12) is the same as Eq. (3), which means Eq. (3) has its explanation based on the phenomenological theory of thermal activation. This explanation also confirms that  $\alpha_1$  and  $\alpha_2$  in Eq. (12) are related to activation energy.

If  $\alpha_1$  and  $\alpha_2$  are treated to be equal as proposed by Maruyama<sup>[4,5]</sup>, Eq.(12) becomes

$$\varepsilon = \varepsilon_0 + A\{1 - \exp(\alpha t)\} + B\{\exp(\alpha t) - 1\}$$
 (13)  
Similarly to Eq. (2), the following equations are proposed in the present paper

$$\ln A = a_1 + b_1 T + c_1 \sigma$$

$$\ln B = a_2 + b_2 T + c_2 \sigma$$

$$\ln C = a_3 + b_3 T + c_3 \sigma$$
(14)

According to Eq. (6) or Eq. (10), it is clear that Q is given by

$$Q = -b_{3}R \tag{15}$$

## 2 Creep of A286 Alloy

If 3 or more measured creep curves of a material are available, all the parameters in Eq. (14) can be determined, and then the creep curve or the creep rupture life of the material at every creep condition can be predicted. As Eq. (13) has been derived in concept of thermal activation, it is necessary to see if the value of Q in Eq. (15) is reasonable in comparison with the activation energy of creep. Besides, it is also important to see if the creep rupture life predicted by Eq. (13) and Eq. (14) can be confirmed by long term creep rupture tests.

The material used was cut from a disc of Fe base 15Cr-26Ni-Mo-Ti-V austenite stainless steel, or A286 as so called. Creep tests were performed under constant stress. Test measured creep curves were fitted by Eq. (13). The results of the fitting are listed in

Table 1. Linear regression was made for all the A, B and  $\alpha$  in Table 1 by Eq. (14), and the result were listed in Table 2.

Table 1 The fitting of measured creep curves of A285 Alloy by Eq.(13)

Test No.	σ/MPa	<i>T /</i> K	A	В	α	<b>E</b> ()
1	657.7	813	2.053	0.142	0.014 5	0.980
2	637.0	813	1.832	0.092	0.009 2	0.850
3	690.9	813	2.206	0.280	0.0254	1.068
4	690.9	793	2.206	0.184	0.008 3	0.606
5	441.0	923	1.398	0.425	0.029 4	0.602
6	441.0	920	1.381	0.248	0.028 0	0.225

Table 2 Parameters of creep equation Eq.(14) for A286 determined by regression

Parameters	$u_{\iota}$	$b_i$	$c_i$
ln A	-5.60	$4.5 \times 10^{-3}$	$4.04 \times 10^{-3}$
ln B	-53.8	$4.7 \times 10^{-3}$	$2.10 \times 10^{-2}$
ln C	25.4	$-3.6 \times 10^4$	$1.98 \times 10^{-2}$

According to Eq. (15) and the value of  $b_s$  in Table 2, Q is  $3.0 \times 10^5$  J/mol. This value is approximate to the activation energy of creep determined by creep rupture tests for the same material [6]. It has been seen in this experiment that Q is very sensitive to the primary stage of creep curves.

Some of the predicted and measured creep curves have been plotted in Fig. 1, which means Eq. (3) can give accurate description and prediction of creep curves.

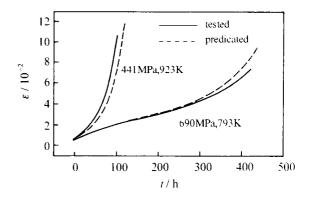


Fig. 1 Some of the measured (solid) and predicted (dash) creep curves of A286 alloy

To predict creep rupture lives by predicted creep curves, it was set for the experiment that

$$\varepsilon_0 = 0.5\%$$

$$\varepsilon_r = 10\%$$

where  $\varepsilon_r$  is the creep rupture strain.

In Fig. 2, creep rupture life is plotted against stress.

The solid lines were calculated, and the triangle symbols were measured by creep rupture tests<sup>[6]</sup>. These two results were in good agreement.

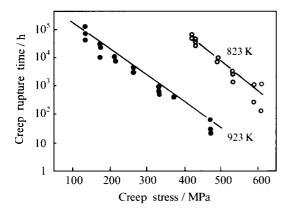


Fig. 2 The creep rupture lives of A286 predicted by Eq.(13) (solid) and test determined (triangle) [6]

#### 3 Discussion

The derivation shown in this paper has given a reasonable explanation about the fact that  $\alpha$  in Eq. (3) was related to creep activation energy<sup>[5]</sup>. It also proposed that  $\alpha$ , or  $\theta_2$  and  $\theta_4$  in Eq. (1), is proportional to the reciprocal of temperature. Evans et al had made a similar derivation<sup>[7]</sup>, but they proposed that  $\theta_2$  and  $\theta_4$  in Eq. (1) are proportional to temperature, as shown in Eq. (2).

The prediction of creep rupture lives gave a good precision in this paper, but Eq. (14), or Eq. (2) as well, is certainly a simplified approximation. In fact, if  $\theta$  projection concept were extrapolated to zero stress, it would give an absurd prediction that the creep strain is not zero when a material is free of stress. Then according to thermal activation theory,  $\alpha_1$  in Eq. (6) and  $\alpha_2$ , in

Eq.(10) become zero when a material is free of stress, so that the creep predicted by Eq.(12) is zero.

#### 4 Conclusion

- (1) The creep equation proposed in  $\theta$  projection concept can be developed in the concept of thermal activation of creep.
- (2) The measured creep curves of A286 alloy haven been fitted by the creep equation. The activation energy involved in the creep equation was calculated, and creep rupture lives were predicted. The result were confirmed by creep data sheet published by NRIM.
- (3) The predicted creep rupture lives made by creep curve prediction based on this creep equation is also confirmed by the creep date sheet published by NRIM.

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