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TIME SERIES AND GREY MODEL DIAGNOSIS METHOD USED TO CRACK PROBLEMS

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ABSTRACT In this paper, the vibration signals in the fatigue crack growth process in a chinese steel used in a mining machinery were analyzed by the frequency spectrum, the time series and grey system model, and the critical criterion for crack initiation was proposed.

KEY WORDS frequency spectrum, time series, grey system model

Recently considerable investigations have been done on the diagnosis of cracks in axles, gear wheels, beams by the vibration method, and great progress has been achieved[1~4], Here, the vibration signals in the fatigue crack growth process in a specimen made of chinese steel used in the mining machinery were analyzed by frequency spectrum, time series and grey system model[5]. Power spectrum, four distance discrimiation funtions of time series, grey system model GM(1, N) are discussed to draw a conclusion for diagnosing crack.

1 FREQUENCY SPECTRUM ANALYSIS OF VIBRATION SIGNALS

The frequency spectrum analysis of the collected vibration signals are carried out in a computer, which experiences more than five steps, i.e., signals output through the recorder meter, D/A transformation sampling, pre-treatment, and FFT process. The power spectrum is calculated by the following formula

$$G_x(f)=(1/n)\sum_{i=1}^{n}|X(f)|^2=(1/n)\sum_{i=1}^{n}X(f)X^*(f) \qquad f=1,\ 2,\ \cdots,\ n \qquad (1)$$

where $X(f)$ is Fourier transform of $x(t)$, and X^* is conjugate variable of $X(f)$.

In order to describe result quantitatively, two new characteristic parameters K_1 and K_2, which show, respectively, the change of base frequency power spectrum magnitude and changing tendency of spectrum value of the base frequency, are introduced. They are

$$K_1=A_i/A_{i0} \qquad i=1,\ 2,\ \cdots,\ 15 \qquad (2)$$

and

$$K_2=A_i/S \qquad i=1,\ 2,\ \cdots,\ 15 \qquad (3)$$

where A_{i0}, A_i are, respectively, the base frequency magnitude values of power spectrum when the length of crack is 0 mm and i mm ($i = 1, 2, \cdots, 15$), S is total energy of the system harmonic wave, K_1 is the ratio of i mm and 0 mm base frequency magnitude values , and K_2 is the ratio of the base frequency energy and total energy.

To find out the changing behaviour of power spectrum because of the crack expanding, two vibration signals are recorded. The power spectrum and magnitude spectrum are calculated when crack length ranges from 0 mm to 15 mm, and the power spectrums under four conditions are chosen. Figure 1 show the power spectrum for the first group of signals, where Fig. 1(a) shows that for the initial condition (no crack), the power spectrum magnitude value of base frequency is smaller than those of two, three, four time frequencies, and energy distributes over multi-frequencies. Fig. 1 (b) ~ (d) respectively depict the spectrums for different conditions.

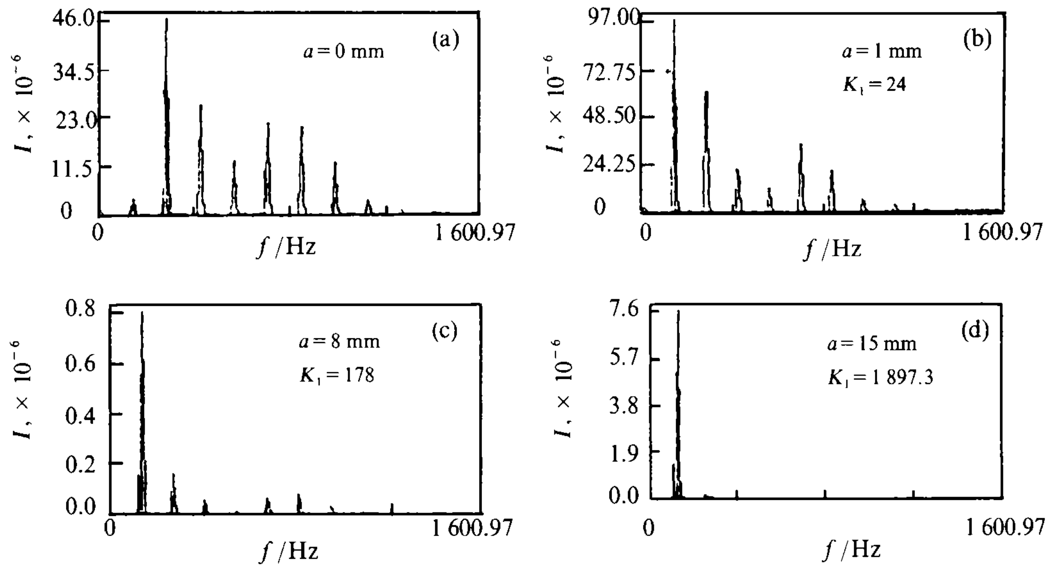

Fig. 1 Power spectrums for four conditions

It is clear that the base frequency magnitude increases when crack grows . Also, the results show that the base frequency value is decreasing with growing of the crack.

Table 1 K_1, K_2, and S of four conditions

Crack length / mm	Base frequency/Hz	Magnitude value	K_1	K_2	S
0	87.8	0.004	1	0.035	0.113
1	86.9	0.096	24	0.43	0.214
8	83.4	1.513	178	0.838	1.805
15	78.2	7.589	1897.3	0.978	7.858

Table 1 shows the calculated results of the base frequency values and the power spectrum magnitude, K_1, K_2 and S. It is easy to see that: (1) when crack growing, the rigidity and base frequency value are dropping, and energy transfers to low frequency; (2) the power spectrum is highly sensitive to crack, sudden change of base frequency magnitude value indicates crack initiating, and magntiude value exponentially increases; (3) K_2 is approaching to 1, which means that the energy of the base frequency is trending to total energy, and the ratio of multi-frequency energy is small.

2 TIME SERIES ANALYSIS

After the vibration signals are samplied and pre-treated, the dispersed time series are obtained. $\{X_k\}$ ($k=1, 2, \cdots, N$) is considered as a stable time series . Then a linear, time feedback time series model could be established

$$X_k - \varphi_1 X_{k-1} - \varphi_2 X_{k-2} - \cdots - \varphi_M X_{k-M} = \alpha_k + \theta_1 \alpha_{k-1} + \cdots + \theta_N \alpha_{k-N} \tag{4}$$

where $\{X_k\}$ is time series which consists of the parameter X_k ($k=1, 2, \cdots, N$); φ_i is autoregression coefficient, from φ_1 to φ_M; θ_i is slide average coefficient, from θ_1 to θ_N; $\alpha_K, \alpha_{K-1}, \cdots, \alpha_{K-N}$ are white noise input, following a normal distribution and independent from each other. Mathematically, α_i could be expressed as: $\alpha_i \sim \text{NID}(0, \delta_a^2)$, which average value is 0, square error is δ_a^2. M, N are, respectively, the order of autoregression and the slide average coefficient. This is called ARMA(M, N) model.

When θ_i is 0 ($i=1,2,\cdots,N$), M autoregression model-AR(M) model can be expressed as follows

$$X_k = \varphi_1 X_{k-1} + \cdots + \varphi_M X_{k-M} + \alpha_k \tag{5}$$

where X_k ($k=1,2, \cdots, M$) is time series; φ_i is autoregression coefficient; and α_k is residual error.

Here, the method used to establish AR model, is based on the Levinson recurrence theory and Burg algorithm, and the FPE (Final Prediction Error) criterion is used to calculate the optimum order of the model.

In order to distinguish two parameter models representing different trouble conditions, it is necessary to find a characteristic parameter describing the difference between two models. To AR model, autoregression cofficient vector $[\varphi]$ can be thought as an N-dimensionl space point, and the reference models constitute the N-dimensional space point system. Then a point to be checked is put in the space, the distance from it to a reference point can be used to verify the consistence between the model to be checked and its corresponding reference model. This function is defined as distance function or discrimination function.

To discriminate the crack condition, four discrimination funtions are mainly studied, which are, respectively, the residual error deviation distance, the Mann distance, the J divergence distance, and the Kullback information distance. In the study, more than four reference models and four models to be checked are given, respectively, corresponding to four crack lengths, 0 mm, 5 mm, 10 mm and 15 mm. The calculated results are shown in Tabs. 2 ~ 5.

Table 2 Residual error deviation distance

Reference model	Checked 1	Checked 2	Checked 3	Checked 4
1	41.87	85.95	74.67	88.25
2	44.46	30.81	94.11	119.92
3	134.19	92.19	71.95	163.64
4	114.35	121.33	170.2	45.80

Table 3 Mann Distance

Reference model	Checked 1	Checked 2	Checked 3	Checked 4
1	4.73	12.57	10.74	29.82
2	5.02	4.51	13.53	40.52
3	15.16	13.48	10.35	55.30
4	16.31	17.74	24.47	15.48

Table 4 Kullback Information Distance $I(P_r, P_t)$

Reference model	Checked 1	Checked 2	Checked 3	Checked 4
1	23.78	62.99	53.02	16.8
2	25.09	22.52	67.66	20.51
3	75.79	67.40	51.77	27.98
4	87.63	88.69	122.32	7.82

Table 5 Kullback Divergence Distance (*J*-Divergence Distance)

Reference model	Checked 1	Checked 2	Checked 3	Checked 4
1	7.00	17.17	45.1	19.0
2	12.1	9.295	21.8	30.0
3	37.87	28.83	19.5	47.9
4	87.63	49.23	60.1	16.5

The data listed in above Tables mean that the smallest of each row tell us the model to be checked is consistent to this reference one. Using these funtions to diagnose the models, we could obtain a result in agreement with the experimental data, which means that the functions are sensitive to crack and can be applied to discriminate the length of crack accurately. Reliablity and effectiveness which establishing AR model and four discrimination funtions diagnoses crack are verified.

We also calculate Green function of AR model, whose convergence is used to discriminate the stablity of system. And Green function is also sensitive to crack, i. e. with crack expanding, Green function slowly attenuate. This is a foundation to diagnose crack [1] .

3 GREY SYSTEM MODEL GM(1,*N*)

We adopt here the grey system differential equations, which could describe the system innate characteristics more better according to the practical demand. This differential equation model is based on interrelative degree convergence principle, AGO (Accumulated Generating Operation) series, grey derivative, grey differential equation. Its characteristics is to find out instinctive law from the stochastic original data[3].

Model GM(1,*N*) is the model of state. It can mirror the effect of $N-1$ variable series on the one-order derivative of dependent variable series. It is one-order linear dynamic state model of *N* series.

(1) Using *N* series X_1, X_2, $\cdots$, X_N to compose the initial model $X_{i(1)}^{(0)}$, $X_{i(2)}^{(0)}$, $\cdots$, $X_{i(N)}^{(0)}$, $i=1, 2, \cdots, N$.

(2) Doing 1−AGO to gain new series: $X_{i(1)}^{(1)}$, $X_{i(2)}^{(1)}$, ⋯, $X_{i(N)}^{(1)}$, $i=1, 2, \cdots, N$.

(3) Establing differential equation:

$$\mathrm{d}X_1^{(1)}/\mathrm{d}t + aX_1^{(1)} = b_1X_2^{(1)} + b_2X_3^{(1)} + b_3X_4^{(1)} + b_4X_5^{(1)} \quad (6)$$

(4) Defining coefficient vector: $\bar{a}=[a, b_1, b_2, \cdots, b_{N-1}]$ (7)

(5) Solving by the least square method: $\bar{a}=(B^{\mathrm{T}}B)-B^{\mathrm{T}}Y_N$ (8)

where B is an accmulator matrix. Y_N is a constant item matrix.

$$B=\begin{bmatrix} -0.5\{X_{(1)1}^{(1)}+X_{(2)1}^{(1)}\} & X_{(2)2}^{(1)} & \cdots & X_{(2)N}^{(1)} \\ -0.5\{X_{(3)1}^{(1)}+X_{(3)2}^{(1)}\} & X_{(3)2}^{(1)} & \cdots & X_{(3)N}^{(1)} \\ \cdots & \cdots & \cdots & \cdots \\ -0.5\{X_{(N-1)1}^{(1)}+X_{(N)1}^{(1)}\} & X_{(N)2}^{(1)} & \cdots & X_{(N)N}^{(1)} \end{bmatrix} \quad (9)$$

Because the values of b_1, b_2, ⋯, b_{N-1} reflect the effects of X_2, X_3, ⋯, X_N on X_1, it can be regarded as a method to diagnose crack condition.

In the paper, we use the above mentioned models which include four reference models and four models to be checked and calculate their power magnitude spectrum values of base frequency, two to five time frequency. The magnitude values of one model to be checked and four reference models are defined as $X_1^{(0)}$, $X_2^{(0)}$, $X_3^{(0)}$, $X_4^{(0)}$, $X_5^{(0)}$ respectively. Then we can obtain four sets of data. (see in Tab. 6)

Table 6 Value of coefficient vector

X_i^*	b_1	b_2	b_3	b_4
Checked model 1	1.13	−0.43	−0.16	0.06
Checked model 2	0.14	0.61	−0.56	0.14
Checked model 3	0.61	−5.3	2.48	0.37
Checked model 4	0.96	−1.13	−0.73	2.47

The bigest value of every row means that model to be checked is consistent to the reference model. This is in agreement with the actual situation.

4 CONCLUSIONS

(1) With crack expanding, rigidity and base frequency value are dropping, and energy transfers to low frequency.

(2) Power spectrum, the residual error deviation distance, the Mann distance, the J divergence distance, and the Kullback information distance are sensitive to crack. They can be used to diagnose initial crack and crack expanding.

(3) Grey system that forecasts crack expanding is feasible and effective.

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时序法和灰色系统理论诊断结构裂纹的研究

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摘要 通过对用于矿山机械的某材料在进行疲劳试验时，测取振动信号，利用频谱、时序方法和灰色系统理论方法对其进行研究，得出诊断裂纹的标准.

关键词 频谱，时序分析，灰色理论