Journal of University of Science and Technology Beijing
Volume 11, Number 3, June 2004, Page 197

Quantitative calculation for the dissipated energy of fault rock burst based on gradient-dependent plasticity

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(Received 2003-05-27)

Abstract: The capacity of energy absorption by fault bands after rock burst was calculated quantitatively according to shear stress-shear deformation curves considering the interactions and interplaying among microstructures due to the heterogeneity of strain softening rock materials. The post-peak stiffness of rock specimens subjected to direct shear was derived strictly based on gradient-dependent plasticity, which can not be obtained from the classical elastoplastic theory. Analytical solutions for the dissipated energy of rock burst were proposed whether the slope of the post-peak shear stress-shear deformation curve is positive or not. The analytical solutions show that shear stress level, confining pressure, shear strength, brittleness, strain rate and heterogeneity of rock materials have important influence on the dissipated energy. The larger value of the dissipated energy means that the capacity of energy dissipation in the form of shear bands is superior and a lower magnitude of rock burst is expected under the condition of the same work done by external shear force. The possibility of rock burst is reduced for a lower softening modulus or a larger thickness of shear bands.

Key words: rock burst; heterogeneity; dissipated energy; plastic strain gradient; post-peak stiffness; characteristic length; fault band; strain softening

[*This work was financially supported by the National Natural Science Foundation of China (No.50309004).*]

1 Introduction

As a kind of sudden and often violent failure of rock in the vicinity of underground excavations, rock burst closely relates to the release of strain energy stored in rock and the dissipated energy. Many observations in field showed that the occurrence of rock burst is related to the present of joints, seams and faults; it is likely to occur adjacent to large-scale faults at great depth. To enrich the understanding of rock burst mechanisms, a further analytical investigation of fault rock burst is necessary.

Since the pioneering work of Cook, from the viewpoint of energy many researchers have studied the mechanisms of rock burst. The important Energy Release Rate (ERR) was firstly calculated to design extraction patterns in mining [1]. The stability analysis of rock specimens and machine systems was carried out by Salamom and a criterion was obtained to predict a stable or unstable failure of mine pillars based on the analogy between laboratory test specimens and mine pillars [2]. To differentiate between the stable failure and the violent one of rock, Salamom proposed a comparison of post-peak stiffness of a coal seam and the loading system (mine roof and floor) [3]. Linkov proposed an energy criterion emphasizing that violent failure occurs when kinetic energy is liberated above that consumed during fracturing of the rock [4]. In addition, to estimate the likelihood of violent failure, several energy indexes were proposed based on stress-strain curves prior to peak stress or complete stress-strain curves under uniaxial compression [5]. Zhang *et al.* and Pan *et al.* proposed the energy criterion of rock burst based on the principle of minimum potential energy and the criterion has been applied to analyze rock burst numerically and analytically [6,7]. Cai *et al.* analyzed the distribution of elastic energy based on numerical simulation by FEM and experimental tests to assess the possibility of rock burst in Linglong gold mine [8,9]. Yin *et al.* proposed a criterion of energy and numerically evaluated the zone prone to rock burst [10]. Li *et al.* discussed the dissipated energy and the released energy after rock burst under uniaxial compression based on complete stress-strain curves [11]. Wang *et al.* analyzed the instability of a direct shear system composed of shear bands and testing

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machines and the instability criterion was proposed based on the principle of minimum potential energy and gradient-dependent plasticity [12]. The criterion can be applied to investigate the static instability of fault rock burst. Considering the effects of strain rate and strain gradient, Wang *et al.* expanded the static analysis because rock burst belongs to a kind of dynamic fracture phenomenon [13].

Though a significant amount of work has been carried out to understand the mechanisms of rock burst and to access the likelihood of rock burst, many problems have not been solved, for example, quantitative calculation for the dissipated energy of rock burst according to stress-strain curves, the effect of specimen size on the dissipated energy, *etc.* The two problems can not be dealt with according to the classical elastoplastic theory whose constitutive relation has no characteristic length determining the size of localized bands.

In the paper, the post-peak stiffness of rock specimens subjected to dynamic shear loading and the dissipated energy of rock specimens for different sizes were calculated quantitatively based on gradient-dependent plasticity.

2 Gradient-dependent plasticity and post-peak shear displacement

Gradient-dependent plasticity has strictly theoretical foundation [14] and it can be derived from early non-local elasticity models proposed by Eringen *et al.* For the one-dimensional shear problem, non-local plastic shear strain depends on both local plastic shear strain and its second-order spatial derivative in internal length, which describes the interactions and interplaying among microstructures [12,13,15-20]. It is noted that when the characteristic length is zero, the gradient term will disappear and the gradient-dependent plasticity will be simplified as the classical elastoplastic theory.

The problem of rock burst is depicted in **figure 1**. The dynamic constitutive relation between shear stress and plastic shear strain is shown in **figure 2**. The horizontal relative shear displacement between the upper and the lower ends of the shear band can be obtained [13]:

$$u_1 = 2\int_0^{w/2} \gamma \mathrm{d}y = \frac{\tau}{G}w + \frac{\tau_c' - \tau}{c'}w \qquad (1)$$

where G is the shear elastic modulus; τ is the post-peak shear stress; c' is the dynamic shear softening modulus, $c' = fc$, f describes the effect of strain rate, $f = 1 + C\ln\gamma_1/\gamma_0$, C is a material constant, γ_1 is the average shear strain rate, γ_0 is the average shear strain rate under quasi-static conditions; $c = G\lambda/(G+\lambda)$ is the absolute value of the slope of static shear stress-plastic shear strain curves; λ is the absolute value of the slope of static shear stress-shear strain curve; τ_c' is the peak shear stress, $\tau_c' = f\tau_c$, τ_c is the static shear strength; γ is the total shear strain of fault band, $\gamma = \gamma^e + \gamma^p(y)$, γ^e is the elastic shear strain, $\gamma^p(y)$ is the local plastic shear strain; y is the coordinate; w is the thickness of fault band, $w = 2\pi l$, and l is the characteristic length of rock material. It is noted that λ and τ_c are dependent on the confining pressure σ.

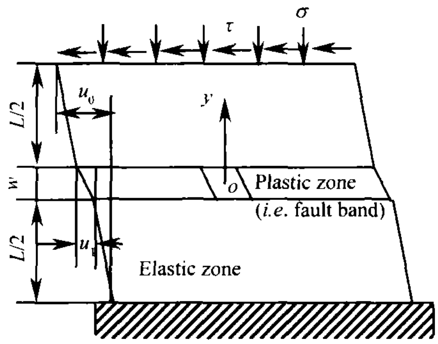

Figure 1 Fault band and elastic zone outside the band.

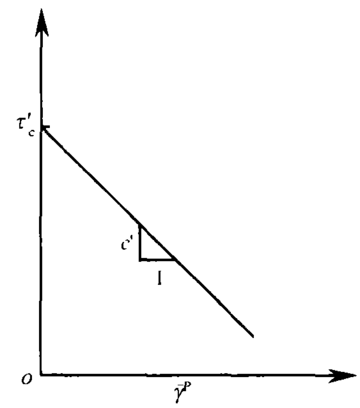

Figure 2 Dynamic softening constitutive relation for fault bands.

3 Post-peak stiffness and analysis of dissipated energy

3.1 Complete shear stress-shear deformation curve

In the elastic stage, the shear localization does not occur and the relation between shear stress and elastic shear strain can be obtained in terms of shear Hooke's law

$$\frac{\mathrm{d}\tau}{\mathrm{d}u_e} = \frac{G}{L+w} \qquad (2)$$

where u_e is the relative elastic shear displacement prior to peak shear stress and L is the size of elastic rock outside the fault band. Outside the band, rock is always elastic or intact whether the shear localization is initiated or not. According to shear Hooke's law, the relative elastic shear displacement outside the band can be written as

$$u_2 = \frac{\tau}{G} L \tag{3}$$

The horizontal relative shear displacement between the upper and the lower ends of the specimen u_0 can be written as

$$u_0 = u_1 + u_2 \tag{4}$$

When considering the expressions of c' and τ_c', substitution of equations (1) and (3) into equation (4) yields

$$u_0 = \frac{\tau}{G}(L + w) + \frac{\tau_c f - \tau}{cf} \cdot w \tag{5}$$

Differentiating both sides of equation (5) with respect to τ, the following expression will be given:

$$du_0 = \left(\frac{w+L}{G} - \frac{w}{cf} \right) d\tau \tag{6}$$

According to equation (6), it can be given as follows:

$$\frac{dF}{du_0} = A \left(\frac{w+L}{G} - \frac{w}{cf} \right)^{-1} \tag{7}$$

where A is the shear area, F is the shear force and $F = A\tau$. It is found that the post-peak behavior of direct shear tests depends on the height of the specimen, as is similar to uniaxial compression for quasi-brittle materials [15,21,22]. Therefore, the slope of post-peak shear stress-shear deformation curves obtained from experiments can not be seen as a constitutive parameter. If the post-peak behavior is steep, then the instability does occur certainly. Contrarily, if the post-peak behavior is not steep, then the specimen will be always in stable state.

Figure 3 shows the relation between shear stress and relative displacement with f=1, G=20 GPa, w=0.04 m, λ=2 GPa and τ_c=20 MPa. Notice that the relative displacement corresponding to the shears strength is higher for a large specimen, as is consistent with experimental results [23]. Increasing the height of the specimen leads to the steeper post-peak behavior, even the snap-back instability.

3.2 Quantitative calculation of energy dissipation for class I and class II behavior

In reference [11], only when the post-peak response exhibits snap-through behavior (*i.e.* the well-known class I behavior in rock mechanics), the dissipated energy in rock fracture was analyzed. But the analysis is qualitative owing to the post-peak stiffness of rock specimens can not be obtained in the context of the classical elastoplastic theory, for the thickness of localized bands is unknown.

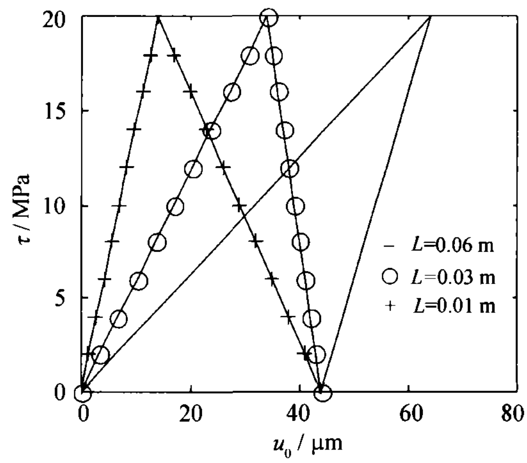

Figure 3 Specimen size effect of shear stress-shear deformation curves from direct shear tests.

In the present analysis, the energy dissipation for the snap-through behavior and the snap-back behavior (*i.e.* class II behavior) will be analyzed quantitatively.

When the post-peak behavior exhibits class I behavior (the slope of the shear force-shear deformation curve is negative $k < 0$) shown in **figure 4** (a), according to equation (7) the snap-through condition is

$$\frac{w+L}{G} < \frac{w}{cf} \tag{8}$$

The absolute value of the slope of the shear stress-shear deformation curve $|k|$ can be expressed as

$$|k| = A \left(\frac{w}{cf} - \frac{w+L}{G} \right)^{-1} > 0 \tag{9}$$

The dissipated energy in rock fracture is the area under the shear force-plastic shear displacement curve. It can be calculated from the following expression:

$$U_s = \frac{F_c^2 (L+w)}{2AG} + \frac{F_c^2}{2|k|} - \frac{F^2 (L+w)}{2AG} - \frac{F^2}{2|k|} \tag{10}$$

where $F_c = A\tau_c'$. Substitution of equation (9) into equation (10) results in the simpler formula:

$$U_s = \frac{w(F_c^2 - F^2)}{2Acf} \tag{11}$$

The elastic zone outside the fault band does not dissipate any energy in elastic or plastic stage and the volume of the localized band where all energy is dissipated is

$$V = Aw \tag{12}$$

The dissipated energy per unit volume can be written as

$$\rho_s = \frac{U_s}{V} = \frac{\tau_c^2 f^2 - \tau^2}{2cf} \tag{13}$$

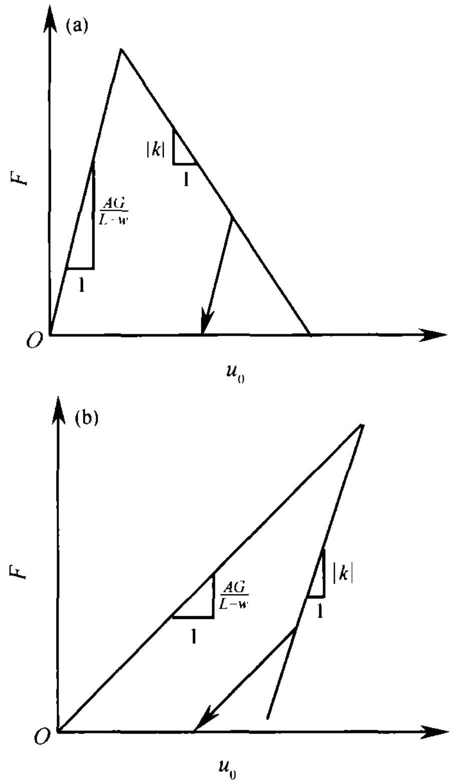

Figure 4 Two kinds of complete shear stress-strain deformation curves: (a) for class I behavior; (b) for class II behavior.

When the post-peak behavior exhibits class II behavior (the slope of the shear force-shear deformation curve is positive, $k > 0$) shown in figure 4(b), according to equation (7) the snap-back condition is

$$\frac{w+L}{G} > \frac{w}{cf} \tag{14}$$

The slope of the shear stress-shear deformation curve can be written as

$$k = A\left(\frac{w+L}{G} - \frac{w}{cf}\right)^{-1} > 0 \tag{15}$$

The dissipated energy can be calculated as according to figure 4(b):

$$U_s = \frac{F_c^2(L+w)}{2AG} - \frac{F_c^2}{2k} - \frac{F^2(L+w)}{2AG} + \frac{F^2}{2k} \tag{16}$$

Substitution of equation (15) into equation (16) yields the following expression:

$$U_s = \frac{w\left(F_c^2 - F^2\right)}{2Acf} \tag{17}$$

It is pointed that equation (17) is identical with equation (11). That is to say, whether the post-peak behavior exhibits snap-through or not, specimens with the same thickness localized band dissipate the same energy in the strain softening stage.

Similar to uniaxial compression tests, the post-peak stiffness of rock specimens subjected to direct shear is size-dependent, so the slope of the shear stress-shear deformation (or strain) curve is not a material property. According to the curves exhibiting class I behavior and class II behavior, the proposed analytical solutions for dissipated energy show that the dissipated energy is related to the flow shear stress, peak shear stress, softening modulus, shear strain rate and volume of the localized band. That is to say, stress level (described by flow shear stress and constant compressive stress), shear strength, brittleness or ductility (reflected by softening modulus), dynamic loading conditions (determined by shear strain rate) and heterogeneity of rock materials (measured by the characteristic length of rock material or the thickness of shear bands) have an important influence on the dissipated energy.

Increasing the shear strength and the shear strain rate as well as the thickness of shear bands and decreasing the softening modulus and flow shear stress will cause the dissipated energy to be increased. The larger value of dissipated energy means that the capacity of energy absorption or dissipation in the form of shear bands is superior and the low magnitude of rock burst is expected under the condition of the same work done by external shear force, for the magnitude of rock burst is in connection with elastic strain energy stored gradually in the entire specimen and released suddenly once rock burst occurs.

Moreover, measures for decreasing the softening modulus or increasing the characteristic length will lead to the impossibility of occurrence of rock burst regardless of the changes in other related parameters. One reason is that lower brittleness can not result in the instability of rock specimens; the other reason is that for rock with lower brittleness the localized band can absorb much of energy than that with higher brittleness, part of which will be converted into other types of energy, such as light, heat and sound. On one hand, the larger characteristic length increases the dissipated energy; on the other hand, the larger the characteristic length, the more stable the specimen is, for the post-peak stiffness of rock specimen is lower.

4 Conclusions

(1) The occurrence of rock burst is strongly related

to the released energy and the dissipated energy. The former can be determined by classical elastic mechanics and the latter can be derived form gradient-dependent plasticity in which interactions and inter-playing among microstructures are considered. It is found that the post-peak stiffness of rock specimens subjected to direct shear is size-dependent. The shear strength, brittleness, strain rate and heterogeneity of rock materials have an important influence on the dissipated energy.

(2) Increasing the shear strength and the shear strain rate as well as the thickness of shear bands and decreasing the softening modulus and flow shear stress cause the dissipated energy increasing. A larger value of the dissipated energy means that the capacity of energy absorption or dissipation in the form of shear bands is superior and a low magnitude of rock burst is expected under the condition of the same work done by external shear force. Measures for decreasing the softening modulus or increasing the characteristic length lead to the impossibility of occurrence of rock burst regardless of the changes in other related parameters.

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