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Dynamic response to road roughness on a tractor-semitrailer system with driver body model

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Abstract: A linear mass-spring system model of a tractor-semitrailer together with driver body parts and sprung seat is presented. Natural frequencies of the system are calculated and response of components in the system to road roughness is completed by means of computer simulation and power spectral density (PSD) approach in all of road conditions and loading cases. The results show that the severest situation of response of the system occurs when the road in rough condition and vehicle unladen. The most sensitive frequency to human body parts is around 0.9 Hz, and model types of a human body seem to be not significant to the response of a heavy tractor-semitrailer system, including to the response of the driver himself.

Key words: tractor-semitrailer; dynamic response; road roughness; computer simulation; power spectral density

For their high efficiency in operation, tractor-semitrailers are popularly used in the world as a sort of important means of cargo transportation. Ride performance of tractor-semitrailers, which is related to driver comfort and freight safety, is one of their major design considerations and has being studied for years as one of prime subjects of vehicle dynamic analysis. Previous investigations on dynamic response of tractor-semitrailers were carried out with the assumption that the driver body was treated as an independent lumped mass in the system and was connected to the tractor with a rigid seat [1-4]. Medical surveys, however, have proved that vibration of trucks or tractors either causes or aggravates a number of disorders of spines and supporting structures of the drivers and also viscera illnesses, and that in order to evaluate the effects of vibration on driver body parts, mechanical simulation of the human body characteristics together with the seat is necessary [5, 6].

In this paper, a linear mass-spring system model of a tractor-semitrailer together with driver body parts and sprung seat is presented. Natural frequencies of the system are calculated and dynamic response of components in the system to road roughness is completed by means of computer simulation and PSD (power spectral density) approach. The result discussion and the modeling comments are given.

1 Modeling

1.1 Tractor-semitrailer model

A tractor-semitrailer is essentially a combination of a tractor and a semitrailer with a kingpin that articulates the two parts into a whole. The vehicle is supported by three sets of suspension and axle-wheel assemblies. Due to the roughness of road, on which the tractor-semitrailer is travelling, vibrations of components of the vehicle are induced.

For the simplicity of modeling, it is assumed that: (1) each of the three suspensions and tires is a linear spring with a parallelly connected viscous damper; (2) each axle is a lumped mass located between the suspension and the tire; (3) both the tractor chassis and the semitrailer platform are rigid bodies with concentrated masses at their own centers of gravity, and pitching inertial moments about the centers of gravity; (4) the kingpin is a spring with very large stiffness in both vertical and roll directions. Because of geometrical symmetry of the vehicle, the symmetric plane of which is its pitching plane, and also its response symmetry, assuming the inputs from left and right wheel tracks are identical both in amplitude and in phase, an idealized model of the tractor-semitrailer may be finally reduced to a two-dimensional mass-spring system having vertical and pitching motions only.

1.2 Human body model

In modeling, a seating driver body in the system could be thought consisting of several lumped masses: head, back, torso, thorax, diaphragm, abdomen, pelvis, and a sprung seat, with each pair of adjacent masses in the model connected by a spring and a parallel damper to represent more exactly the elastic and damping properties of human body tissue as well as the seat suspen-

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sion. Parameter values of a driver-seat model can be found in reference [6].

1.3 Combined model

A combined linear mass-spring system model of a tractor-semitrailer with human body is shown in **figure 1**.

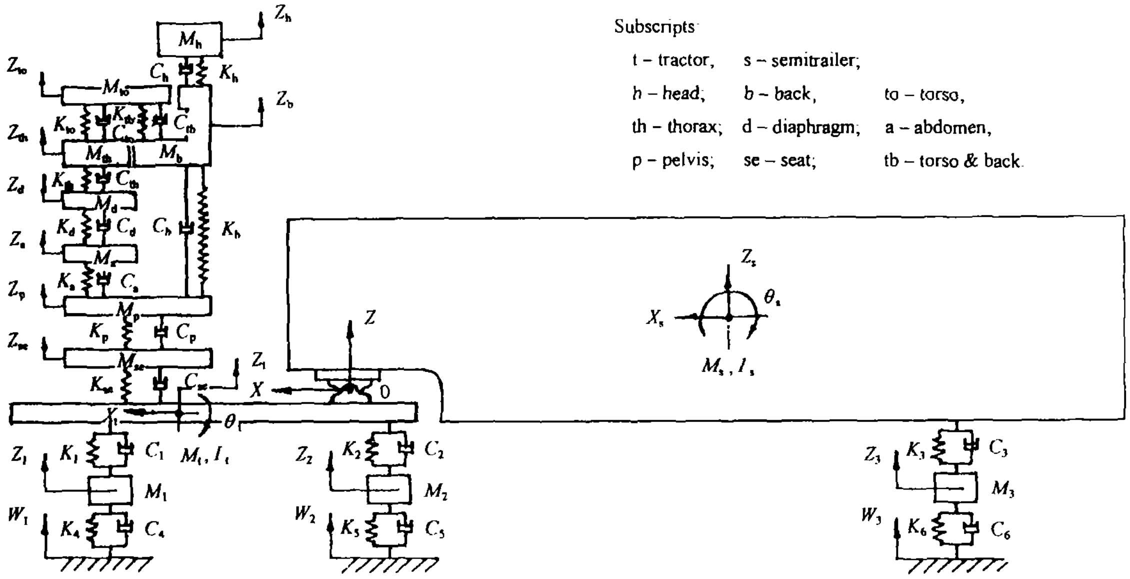

Figure 1 A tractor-semitrailer model with driver body parts (symbols are listed in table 1).

2 Motion equation

For the linear mass-spring system under consideration, the motion equations can be expressed by the following matrix form:

$$\boldsymbol{M}\ddot{\boldsymbol{Z}}+\boldsymbol{C}\dot{\boldsymbol{Z}}+\boldsymbol{K}\boldsymbol{Z}=\boldsymbol{F}(t) \qquad (1)$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ are the mass matrix, damping coefficient matrix, and stiffness constant matrix of the system, respectively; $\boldsymbol{F}(t)$ is the input force vector to the system, and $\boldsymbol{Z}$, $\dot{\boldsymbol{Z}}$, $\ddot{\boldsymbol{Z}}$ are the generalized displacement, velocity, and acceleration vectors of the system, respectively.

The individual equation of motion in equation (1) and the matrices $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ can be derived from the Lagrangian Equation. Solutions of equation (1) can be obtained by PSD approach.

3 PSD approach

The PSD approach is the most useful technique in frequency domain analysis for linear dynamic systems excited by steady random inputs. A random process, such as road roughness, can be decomposed by Fourier Transform procedure into a series of sine waves of different wavelengths. The sine waves vary in their amplitudes and phase relationships. PSD is a plot of these amplitudes versus spatial frequencies. It can hence clearly illustrate the amplitude contents, as functions of frequencies, of the random process. Applying PSD analytical technique, the relation between output random responses and input random signals can be established by a simple mathematical expression. Taking it into consideration, for a tractor-semitrailer system, the PSD function matrix of output displacements, $\boldsymbol{S}_Z(f)$, can be expressed in terms of the PSD function matrix of input forces, $\boldsymbol{S}_F(f)$, as the following form [2,7]:

$$\boldsymbol{S}_Z(f)=\boldsymbol{H}^*(f)\,\boldsymbol{S}_F(f)\,\boldsymbol{H}^T(f) \qquad (2)$$

and the output acceleration PSD function matrix, $\boldsymbol{S}_A(f)$, is determined by the following equation [8]:

$$\boldsymbol{S}_A(f)=(2\pi f)^4\boldsymbol{S}_Z(f) \qquad (3)$$

The diagonal terms of matrix $\boldsymbol{S}_Z(f)$ represent the PSD of vector $\boldsymbol{Z}$, while off-diagonal terms the cross spectral densities. To evaluate mean square values of displacement vector $\boldsymbol{Z}$, we need just integrate the diagonal elements of matrix $\boldsymbol{S}_Z(f)$ over effective frequency band width.

The $\boldsymbol{H}(f)$ in equation (2) is a frequency response function matrix of the system, which is also called Transfer Function Matrix in a linear-elastic system. The $\boldsymbol{H}(f)$ can be derived from equation (1) by the following steps:

$$[\boldsymbol{K}-(2\pi f)^2\boldsymbol{M}+\mathrm{i}2\pi f\boldsymbol{C}]\boldsymbol{Z}=\boldsymbol{F}(t)$$

$$\boldsymbol{H}(f)=\boldsymbol{Z}/\boldsymbol{F}(t)=[\boldsymbol{K}-(2\pi f)^2\boldsymbol{M}+\mathrm{i}2\pi f\boldsymbol{C}]^{-1} \qquad (4)$$

The input force PSD function matrix, $\boldsymbol{S}_F(f)$, is determined by the road characteristics and the elastic and damping properties of tires.

4 Road excitation

Road roughness, which is the excitation source to the tractor-semitrailer system and is treated as a random process, can be expressed in terms of PSD as [9]:

$$\boldsymbol{S}(n)=\begin{cases}\boldsymbol{S}(n_0)(n/n_0)^{-r_1}\ , & (n\le n_0)\\ \boldsymbol{S}(n_0)(n/n_0)^{-r_2}\ , & (n\ge n_0)\end{cases} \tag{5}$$

In equation (5), n is spatial frequency of the road roughness, n_0 is discontinuity frequency between two branches of equation (5), $\boldsymbol{S}(n)$ is spectral density function of the roughness, $\boldsymbol{S}(n_0)$ is roughness coefficient (the spectral density value at the discontinuity frequency n_0), and r_1, r_2 are waviness exponents.

For convenience of computation, the spatial frequencies n and n_0 in equation (5) should be converted into temporal ones. When the tractor-semitrailer is travelling at a constant velocity V, it is done just by changing the spatial coordinate X to $V\cdot t$, where t is time. Thus, the road input PSD function, equation (5), becomes:

$$\boldsymbol{S}(f)=\begin{cases}\boldsymbol{S}(f_0)(f/f_0)^{-r_1}\ , & (f\le f_0)\\ \boldsymbol{S}(f_0)(f/f_0)^{-r_2}\ , & (f\ge f_0)\end{cases} \tag{6}$$

where f, f_0, $\boldsymbol{S}(f_0)$ and $\boldsymbol{S}(f)$ correspond to n, n_0, $\boldsymbol{S}(n_0)$ and $\boldsymbol{S}(n)$, respectively, in which $f_0=n_0V$ and $\boldsymbol{S}(f_0)=\boldsymbol{S}(n_0)/V$. The road surface characteristic parameters for both smooth road and rough road can be found in reference [1]. Equation (6) is illustrated in **figure 2**.

In the case of travelling at a constant velocity on a rough road, the tractor-semitrailer would have three vertically spatial displacements imposed by its three wheel sets with a time delay between each two. Assuming L_{jk} is the spacing between the jth wheel and the k th wheel, the cross-correlation function of inputs at the jth wheel and the kth wheel is given by:

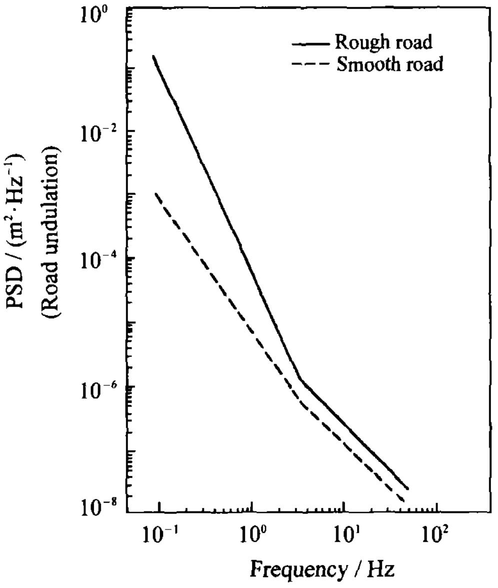

Figure 2 Input power spectral density.

$$R_{jk}(\tau)=R_{jj}(\tau-L_{jk}/V) \tag{7}$$

and hence the cross spectral density of inputs at the two wheels has the following form:

$$\begin{aligned}\boldsymbol{S}_{jk}(f)&=\int_{-\infty}^{\infty}R_{jj}(\tau-L_{jk}/V)\,\mathrm{e}^{-\mathrm{i}2\pi f\tau}\,\mathrm{d}\tau\\ &=S_{jj}(f)\exp(-\mathrm{i}2\pi fL_{jk}/V)\end{aligned} \tag{8}$$

Thus, the input displacement PSD matrix becomes:

$$\boldsymbol{S}_{\mathrm{W}}(f)=\boldsymbol{S}(f)\begin{bmatrix}1 & \mathrm{e}^{-\mu_{12}} & \mathrm{e}^{-\mu_{13}}\\ \mathrm{e}^{\mu_{21}} & 1 & \mathrm{e}^{-\mu_{23}}\\ \mathrm{e}^{\mu_{31}} & \mathrm{e}^{\mu_{32}} & 1\end{bmatrix} \tag{9}$$

where $\mu_{jk}=\mu_{kj}=2\pi fL_{jk}/V$.

Therefore, the input force PSD matrix $\boldsymbol{S}_{\mathrm{F}}(f)$ is obtained by means of the properties of tires as follows [2]:

$$\boldsymbol{S}_{\mathrm{F}}(f)=(\boldsymbol{K}_{\mathrm{F}}+\mathrm{i}2\pi f\boldsymbol{C}_{\mathrm{F}})^{*}\boldsymbol{S}_{\mathrm{W}}(f)(\boldsymbol{K}_{\mathrm{F}}+\mathrm{i}2\pi f\boldsymbol{C}_{\mathrm{F}})^{\mathrm{T}} \tag{10}$$

where $(\boldsymbol{K}_{\mathrm{F}}+\mathrm{i}2\pi f\boldsymbol{C}_{\mathrm{F}})$ is a matrix of 3 by 3 orders with diagonal terms equal to $(k_m+\mathrm{i}2\pi fc_m)$, $m=4$, 5 and 6, and all of the off-diagonal terms equal to zero.

5 Results and discussion

5.1 Natural frequencies

For a particular tractor-semitrailer system, natural frequencies were computed based on the model shown in figure 1 and equation (1) with damping ignored. These frequencies, in both unladen and laden cases, with their dominant terms in corresponding mode shapes, are given in **table 1**.

There are 14 natural frequencies in this system which has 14 degrees of freedom. In unladen case, f_1, f_2, f_3, f_6, f_{10}, f_{12} and f_{14} are the natural frequencies at which the vertical motions of some human body parts (abdomen, thorax, diaphragm, head and back) are dominant in mode shapes. It means that these frequencies in the range from 0.871 45 to 26.756 55 Hz are sensitive to human body. In laden case, the resonant frequency range is about the same as that in unladen case, but the first five lowest frequencies are reduced and the relevant mode shapes are also changed. Besides, the dominant term in mode shape of the fourth natural frequency, f_4, is moved from rear axle bounce to abdomen vertical motion. All these variations are caused by the changes of the mass and inertial moment of the semitrailer, as well as of some dimensions of the vehicle.

5.2 Response

The responses of components in the system to different road roughness expressed by equation (6) with different loading cases are expressed in forms of acceleration PSD curves. **Figures 3 and 4** show the PSD curves of vertical accelerations on the tractor of the system

Table 1 Natural frequencies and dominant terms

Hz

Natural frequencies	Loading cases		Dominant terms in mode shapes
	Unladen	Laden	
f_1	0.871 45	0.870 85	Z_a—abdomen vertical motion
f_2	1.507 38	1.275 01	Z_a—abdomen vertical motion
f_3	2.272 08	1.527 64	Z_a—abdomen vertical motion
f_4	2.563 61	—	Z_3—rear axle bounce
f_4	—	2.275 62	Z_a—abdomen vertical motion
f_5	3.418 41	2.510 44	θ_t—tractor pitch
f_6	4.822 51	4.821 77	Z_{th}—thorax vertical motion
f_7	6.567 22	6.527 26	Z_{se}—seat vertical motion
f_8	10.176 63	10.147 45	Z_3—rear axle bounce
f_9	10.202 82	10.164 67	Z_2—middle axle bounce
f_{10}	10.524 19	10.528 36	Z_d—diaphragm vertical motion
f_{11}	11.654 50	11.649 48	Z_1—front axle bounce
f_{12}	12.339 87	12.339 84	Z_h—head vertical motion
f_{13}	13.798 34	13.798 42	Z_{se}—seat vertical motion
f_{14}	26.756 55	26.756 45	Z_b—back vertical motion

to rough and smooth roads for unladen and laden cases, respectively. These curves are just as the same as those shown in references [1] and [7] in which the driver body is assumed as a lumped mass with rigid seat. Comparing each pair of curves in the same figure, figure 3 or figure 4, with each other, the only difference between which is the road condition, it can be seen that the shapes of them are similar and they have the same frequency locations of peaks, but their amplitudes at any frequency are quite different: larger in rough road and smaller in smooth road. Comparing figure 3 with figure 4, we find that the responses to vertical motions of the tractor in laden case, in general, have smaller values in amplitudes and are a little smoother in shapes than the curves in unladen case. From this we come to the point that the severest situation of these motions emerges under rough road condition and in unladen case, which should be considered in vehicle design.

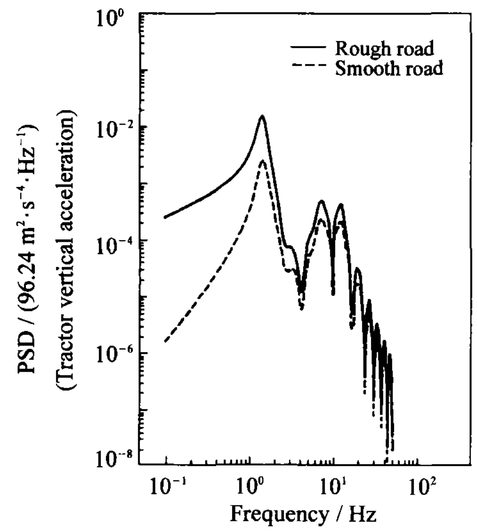

Figure 3 Output power spectral density (unladen).

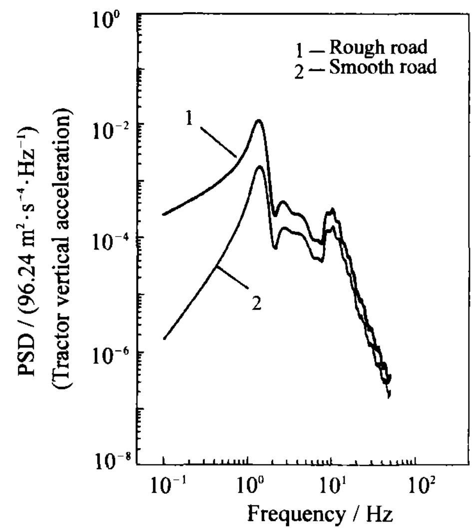

Figure 4 Output power spectral density (laden).

Figures 5-8 present vertical acceleration PSDs on some of driver body parts: pelvis, thorax, back and head respectively, in unladen case and under rough road condition. Comparing these curves with each other, we can see clearly that they have almost the same shapes and the same frequency locations of peaks, even coincide with each other in most of the frequency range, except for little differences at some peak amplitude values around frequencies of 3.5, 9.85, 10.5 and 11.1 Hz, which are close to the natural frequencies of f_5, f_8, f_{10} and f_{11} of the system. It should be noticeable that the maximum amplitude peak values in all of these curves

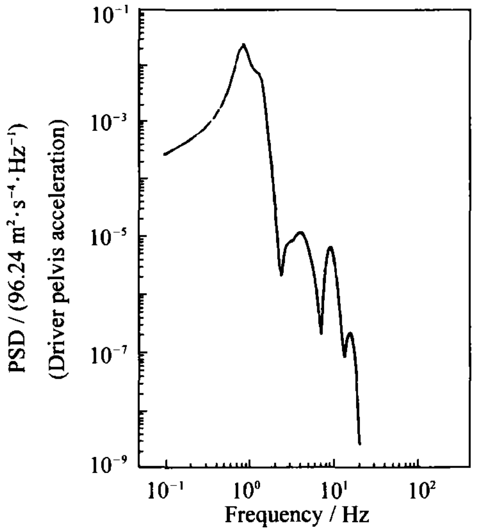

Figure 5 Output PSD at driver pelvis (rough road, unladen).

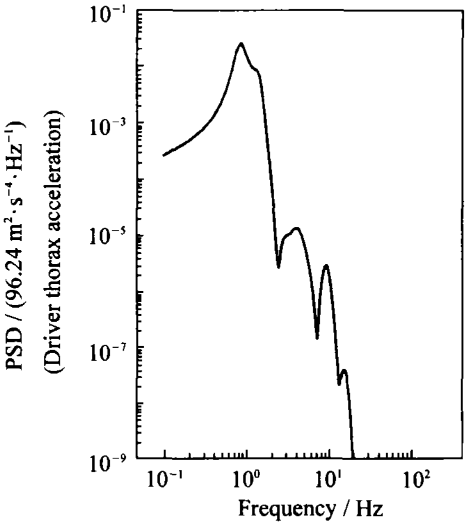

Figure 6 Output PSD at driver thorax (rough road, unladen).

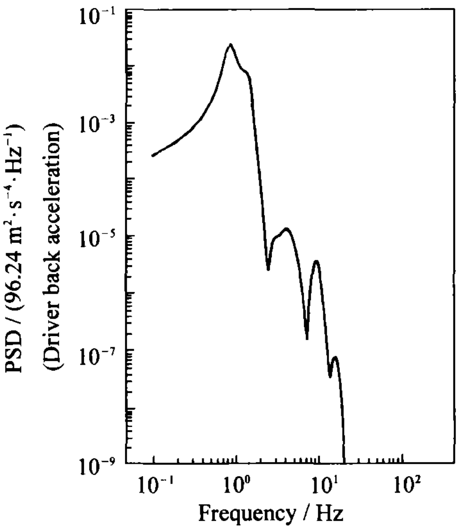

Figure 7 Output PSD at driver back (rough road, unladen).

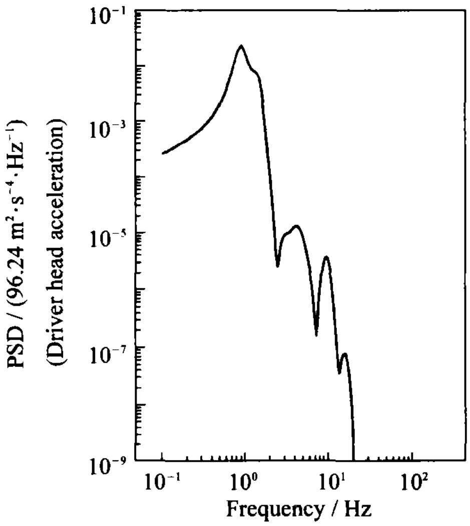

Figure 8 Output PSD at driver head (rough road, unladen).

occur at around 0.9 Hz which is very close to the natural frequency f_1. It tells us that the lowest natural frequency f_1 of the system has major influence on the responses of human body parts.

The curve in **figure 9** shows the response of the driver seat. The resonant amplitudes of this curve occur at the same frequency locations as shown in figures 5-8, and amplitude values in frequency range over 3.5 Hz are larger than those in previous figures. It means that the responses of driver body parts are smaller than that of the seat. In other words, the assumed tissue springs in the human body model exert the function of isolation, which reduces the vibration suffered by the driver from the seat in higher frequency range.

More over, when the driver body is assumed as a lumped mass with the same seating condition, its response, as **figure 10** shows, is extremely similar with the responses of some of driver body parts shown in figures 5-8 , especially in the lower frequency range. It indicates again that the model types of a driver body in a heavy tractor-semitrailer system is not significant to

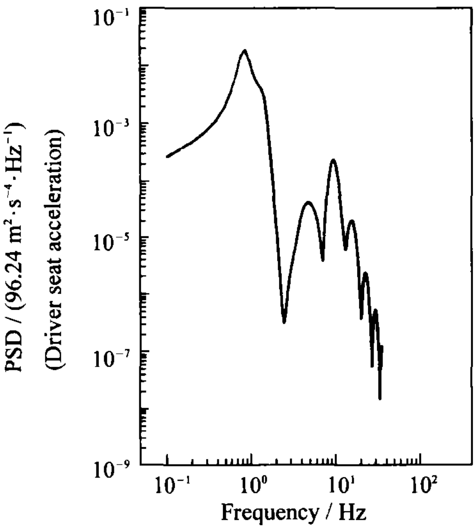

Figure 9 Output PSD at driver seat (rough road, unladen).

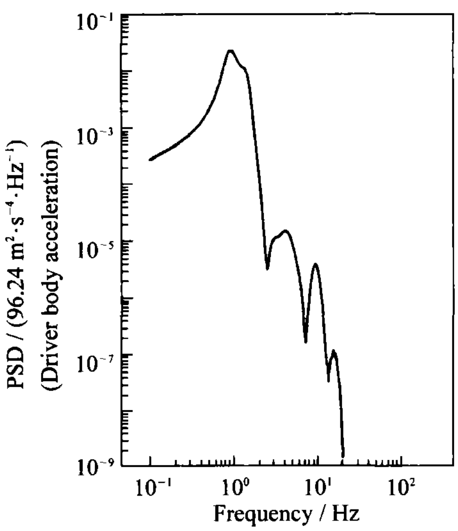

Figure 10 Output PSD at driver body (rough road, unladen).

the responses, not only of vehicle components, but also of human body parts.

6 Conclusions

(1) The road conditions and vehicle loading cases do influence the response of a tractor-semitrailer system. The severest situation of response occurs when road under rough condition and vehicle in unladen case.

(2) The tractor-semitrailer system model, in which the driver body with seat is assumed as a linear mass-spring subsystem, is reasonable. The frequency range of road excitation from 0.871 45 to 26.876 55 Hz is sensitive to the responses on human body parts and the most sensitive frequency is around 0.9 Hz.

(3) Although good results are obtained from the presented model, it seems that model types of a driver body, as a linear mass-spring subsystem or a lumped mass, in a heavy tractor-semitrailer system is not significant to the responses of both vehicle components and human body parts. The sprung seat, however, due to its isolating function, can reduce the vibration suffered by the driver.

(4) Applying PSD approach to analyze dynamic response to a steady random process is effective. The results obtained in this paper for a tractor-semitrailer system are of great significance to the judgement and evaluation of ride comfort of the vehicle, to the strength analysis of its structure, and also to its optimal design. The approach applied and results obtained are suitable to dynamic analysis of all sorts of road vehicles as well.

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