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A Constructive Method of Pandiagonal Magic Squares*

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Abstract: By means of this approach, a constructive method of pandiagonal magic squares is proposed. Pandiagonal magic squares of order mn can be generated via two ones which are orders m and n, respectively.
Key words: magic square, pandiagonal magic square, magic sum

A magic square of order n is a square matrix consisting of consecutive numbers from 1 to n^2 so that the sums of the elements of each row, each column and each of the main diagonals are the same (it is $n(n^2+1)/2$, called magic sum)[1,2].

In this paper, $[s]$ denotes the congruence class of s modulo n, in particular, $[kn]=n$.

Suppose $A=(a_{ij})$ is an $n\times n$ matrix. For each j, a set of elements

$$a_{i[j-i+1]} \qquad (i=1,2,\cdots,n)$$

or

$$a_{i[j+i-1]} \qquad (i=1,2,\cdots,n)$$

is called a set of pandiagonal elements (hereafter called pandiagonal).

Especially if $j=n$, the pandiagonals becomes

$$a_{1n}, a_{2(n-1)}, \cdots, a_{n1}$$

or

$$a_{11}, a_{22}, \cdots, a_{nn}$$

They are usual diagonals.

A magic square is called pandiagonal if not only it is a magic square but also the sum of the elements of each pandiagonal equals the magic sum[1,3,4].

In section 1 of this paper, a new method to construct pandiagonal magic squares of order mn by ones of orders m and n is set up, and extend the method to construct general magic squares. In section 2, two examples are given.

1 Methods

Theorem 1. Let $A=(a_{ij})$ and $B=(b_{ij})$ be pandiagonal magic squares of orders m and n respectively, $C=(c_{ij})$ a matrix where

$$c_{ij}=n^2(a_{ij}-1)D+B \qquad (i,j=1,2,\cdots,m)$$

with D being a matrix of order n such that its elements are all 1. Then C is a pandiagonal magic square of order mn.

Proof. If relaxing the restrictions, we admit that a magic square of order n consists of n^2 distinct integers. Then each c_{ij} is clearly a pandiagonal magic square of order n. Now let us show that the sums of the elements of each row, each column and each pandiagonal are the same. The proof is proceeded via two steps.

Step 1. Let us prove that the sums of the elements of each row and each column equal the same number (magic sum). Let

$$c_{ij}=(c_{ij}^{hk})$$

then from the definition of C, we have

$$c_{ij}^{hk}=(a_{ij}-1)n^2+b_{hk}\ .$$

Since

$$\sum_{j=1}^{m}a_{ij}=\frac{m(m^2+1)}{2},\quad \sum_{k=1}^{n}b_{hk}=\frac{n(n^2+1)}{2},$$

the sum of the elements of the row in C corresponding to row h in c_{ij} is

$$\sum_{j=1}^{m}\sum_{k=1}^{n}c_{ij}^{hk}=\sum_{j=1}^{m}\sum_{k=1}^{n}[(a_{ij}-1)n^2+b_{hk}]=$$

$$\sum_{j=1}^{m}\sum_{k=1}^{n}(a_{ij}-1)n^2+\sum_{j=1}^{m}\sum_{k=1}^{n}b_{hk}=$$

$$n^3(\sum_{j=1}^{m}a_{ij}-m)+m\sum_{k=1}^{n}b_{hk}=$$

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$$n^3[\frac{m(m^2+1)}{2} - m] + m\frac{n(n^2+1)}{2} = \frac{mn}{2}[(mn)^2+1]$$

The proof about columns is similar to the case above.

Step 2. We prove that the sums of the elements of each pandiagonal equal the foregoing numbers.

First, if a pandiagonal consists of diagonals of c_{ij}, then the sum of its elements equals the foregoing number. Since if the pandiagonal consists of diagonals of

$$c_{1,j}, c_{2,j-1}, \cdots, c_{j,1}, c_{m-1,j+2}, \cdots, c_{j+1,m}$$

the sum of its elements is

$$n^3(a_{1j} + a_{2j-1} + \cdots + a_{m,j+1} + a_{m-1,j+2} + \cdots + a_{j+1,m}) + m\frac{n(n^2+1)}{2} - n^3 m = \frac{mn((mn)^2+1)}{2}$$

The proof in another direction is the same.

Now, let us consider other pandiagonals. Denote

$$A_{ij} = (a_{ij} - 1)n^2 D$$

then C can be written as

$$\begin{bmatrix} A_{11} & \cdots & A_{1m} \\ \cdots & \cdots & \cdots \\ A_{m1} & \cdots & A_{mm} \end{bmatrix} + \begin{bmatrix} B & \cdots & B \\ \cdots & \cdots & \cdots \\ B & \cdots & B \end{bmatrix}$$

Let a pandiagonal start from the element in $(h, 1)$ of A_{ij} and towards the right and down. The sum of the elements in this pandiagonal equals that of the corresponding sum of the elements of

$$\tilde{A} = \begin{bmatrix} A_{11} & \cdots & A_{1m} \\ \cdots & \cdots & \cdots \\ A_{m1} & \cdots & A_{mm} \end{bmatrix}$$

plus the corresponding sum of the elements of

$$\tilde{B} = \begin{bmatrix} B & \cdots & B \\ \cdots & \cdots & \cdots \\ B & \cdots & B \end{bmatrix}$$

If $h>1$, the considered pandiagonal of which starts from the element in $(h, 1)$ of A_{1j} will enter into and $A_{1,j}, A_{1,j+1}, A_{2,j+1}, A_{2,j+2}, \cdots, A_{n-j+2,1}$ and $A_{n-j+1,1}$ (i. e., $A_{i[j+i-1]}(i = 1,2,\cdots,n)$) successively. There are $n - h + 1$ elements of this pandiagonal in $A_{1j}, A_{2j+1},\cdots,A_{n-j+2,1}$ and h-1 elements in $A_{1,j+1}, A_{2,j+2},\cdots,A_{n-j+1,1}$. From the construction of A_{ij}, the sums of elements in these two kinds of lines are

$$(n-h+1)n^2[(a_{1j} + a_{2j+1} + \cdots + a_{n-j+1,n} + a_{n,j-1} + a_{n-1,j-2} + \cdots + a_{n-j+2,1}) - n] = (n-h+1))n^2[\frac{m(m^2+1)}{2} - m]$$

and

$$(h-1)n^2[(a_{1,j+1} + a_{2,j+2} + \cdots + a_{n-j+1,1}) - n] = (h-1)n^2[\frac{m(m^2+1)}{2} - m]$$

respectively.In foregoing derivation, that the sum of elements in a pandiagonal of A equals $m(m^2 + 1)/2$ is used. To plus the two formulas, we obtain that the sum of elements in $\tilde{A}$ corresponding to considered pandiagonal is $n^3[m(m^2+1)/2 - m]$.

Next, let us consider the corresponding pandiagonal of B. Similarly, this pandiagonal will enter into the $(1, j)$ submatrix (write B_{1j}, the others are on the analogy of this), then $B_{1,j+1}\cdots$. Since each B_{ij} is B , the parts in B_{1j} and in $B_{1,j+1}$ form a pandiagonal of B. Consequently the sum of their elements is $n(n^2 + 1) / 2$. This pandiagonal enters such submatrices m times, hence the sum of its elements is $mn(n^2 + 1) / 2$. Since

$$n^3\left[\frac{m(m^2+1)}{2} - m\right] + m\cdot\frac{n(n^2+1)mn}{2\quad 2} = [(mn)^2+1]$$

It equals the preceding number. The proof in another direction is the same. This completes the proof.

Theorem 2. Let $A = (a_{ij})$ be a magic square of order m, $B_{ij}(i, j = 1,2,\cdots,m)$ be magic squares of order n and $C = (c_{ij})$ a matrix where

$$c_{ij} = n^2(a_{ij} - 1)D + B_{ij} \qquad (i,j = 1,2,\cdots,m).$$

Then C is a magic square of order mn.

Proof. It is the same as those stated in theorem 1.

2 Examples

Example 1. By using the method of the reference [3], we can construct a pandiagonal magic square of order 5 as follows

1	7	13	19	25
14	20	21	2	8
22	3	9	15	16
10	11	17	23	4
18	24	5	6	12

Let A, B be both this square, by means of proceeding method a pandiagonal magic square of order 25 can be generated.

Example 2. Let

$$A=\begin{vmatrix} 4 & 5 & 10 & 15 \\ 9 & 16 & 3 & 6 \\ 7 & 2 & 13 & 12 \\ 14 & 11 & 8 & 1 \end{vmatrix}$$

$$B=\begin{vmatrix} 1 & 3 & 54 & 56 & 25 & 27 & 46 & 48 \\ 2 & 4 & 53 & 55 & 26 & 28 & 45 & 47 \\ 31 & 29 & 44 & 42 & 7 & 5 & 52 & 50 \\ 32 & 30 & 43 & 41 & 8 & 6 & 51 & 49 \\ 40 & 38 & 19 & 17 & 64 & 62 & 11 & 9 \\ 39 & 37 & 20 & 18 & 63 & 61 & 12 & 10 \\ 58 & 60 & 13 & 15 & 34 & 36 & 21 & 23 \\ 57 & 59 & 14 & 16 & 33 & 35 & 22 & 24 \end{vmatrix}$$

We can obtain a pandiagonal magic square of order 32.

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Determination of CaO in Baotou Columbite and Steel Cinder with Flame Atomic Absorption Spectrometry

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Abstract: The determination of CaO content in columbite and steel cinder with flame atomic absorption spectrometry is studied. EDTA+TEA is used to eliminate the interferences, in HCl media, with La as releaser. The methods of sample treatment and the CaO in remainder undissolved in acids have been conducted. The result of the determination and recovery of CaO shows that the rate of recovery is 100% ~ 102%, R.S.D<2%.

Key words: atomic absorption spectrometry, CaO, columbite, steel cinder

Dissolution Equilibrium of Bismuth Vapor in Liquid Iron and the Interaction Effect of Third Element

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Abstract: The dissolution equilibrium of Bi vapor in liquid iron and the interaction effect of third element were conducted in a sealed Mo reaction chamber by vapor pressure method. The relationship between the standard solution Gibbs free energy of Bi in liquid iron and temperature obtained can be expressed. The interaction coefficients of third elements on Bismuth in liquid iron at 1 873 K can be deduced.

Key words: bismuth, liquid iron, thermodynamic parameters, impurity

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