

## Fractal Structure of the Core Loss Spectrum of Co-based Amorphous Soft Magnetic Alloy with Constant Permeability

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**Abstract:** In order to check the traditional core loss formula, the core loss spectrum  $P(f)$  of Co-based amorphous soft magnetic alloy with constant permeability has been studied. It is found that within a high frequency range from 10 kHz to 200 kHz and at  $B_m = 0.1$  T, the  $P(f)$  has the fractal structure  $P(f) = P_0 f^{D_f}$ , and with the increasing of induced anisotropy energy  $K_u$ , the fractal dimension  $D_f$  rises, thus the total power loss at high frequency increases and the frequency characteristic of  $P(f)$  becomes worse.

**Key words:** amorphous soft magnetic alloy; constant permeability; core loss; fractal

Traditional constant permeability materials are crystalline alloys. When working at high frequency, however, crystalline alloy has disadvantages—the total power loss is rather great and the stability of its permeability decreases. Recently, Zhang and Li *et al.* have developed Co-based amorphous soft magnetic alloys with constant permeability [1]. Compared with crystalline alloy, amorphous alloy has much lower loss, more stable constant permeability  $\mu$  and better quality factor  $Q$ . The permeability of three kinds of amorphous alloys developed by Zhang *et al.* is from 620 to 2880 H/m. Generally, these alloys work at high magnetization frequency in the range of 10 ~ 200 kHz and with a magnetic induction  $B_m = 0.1$  T. Under the above working conditions, the total power loss spectrum is traditionally given by the following separation formula [2]:

$$P(f) = P_e + P_h + P_c = eB_m^2 f^2 + \eta B_m^{1.6} f + P_c \quad (1)$$

where  $P_e = eB_m^2 f^2$  is the eddy-current loss,  $P_h = \eta B_m^{1.6} f$  the magnetic hysteretic loss, and  $P_c$  the residual loss. Through an assumed physical model of technology magnetization, the loss coefficient  $e$  and  $\eta$  can be calculated by using specific material's properties (electric resistivity and thickness, *etc.*) and magnetic domain structure.

Many experimental results have proven that formula (1) is not suitable for high permeability soft magnetic alloy at high frequency. Their total loss per cycle  $P(f)/f$  can not be simply separated.

In this paper, the loss spectrum  $P(f)$  of three kinds of Co-based amorphous constant permeability alloys

has been investigated in a high frequency range as  $B_m = 0.1$  T. It is found that there is a non-scale frequency range of  $P(f)$  and  $P(f)$  has a typical fractal structure within this range [3]. Furthermore, the influence of anisotropy  $K_u$  induced by transverse magnetic heat treatment on the fractal dimension of  $P(f)$  is discussed.

### 1 Experimental

The chemical compositions of the specimens were shown in table 1. The specimens were annealed at

Table 1 Composition of specimens (mass fraction) %

specimen	Co	Fe	Ni	Si	B
a	64	8	10	6	12
b	64	10	2	14	8
c	52	20	8	7	13

different temperatures (a. 315 °C, b. 340 °C, c. 370 °C) in transverse magnetic field for 3 h. The specimens were thin sheet (0.05 mm in thickness and 10 mm in width) rolled cores with 16mm in internal diameter and 26 mm in external diameter. The anisotropy energy  $K_u$  and D. C. magnetic properties were measured by magnetization work and ballistic methods respectively. The total power loss  $P(f)$  and dynamic permeability as  $B_m = 0.1$  T were tested with a transformer-bridge.

### 2 Experimental Results and Discussions

#### 2.1 Fractal formula of $P(f)$

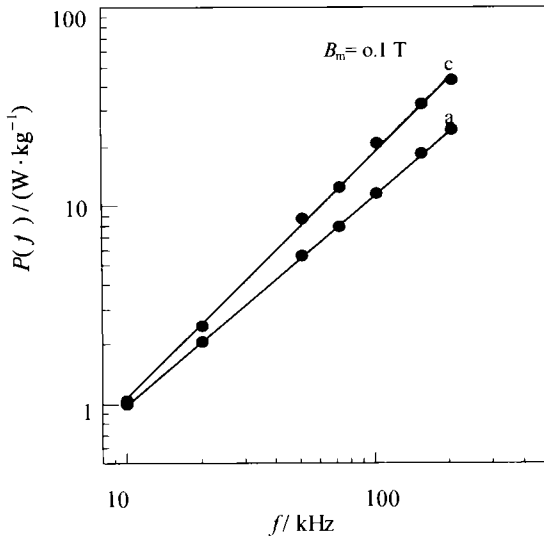
The  $P(f)$  data of specimen a, b, and c measured as

$B_m = 0.1$  T are listed in table 2.

**Table 2 Total power loss  $P(f)$  of specimens ( $B_m=0.1$  T)**

$f / \text{kHz}$	$P / \text{W} \cdot \text{kg}^{-1}$		
	a	b	c
10	1.00	0.85	1.05
20	2.08	2.01	2.50
50	5.70	6.30	8.92
70	8.20	9.70	13.00
100	11.90	15.00	21.60
150	18.90	25.00	33.50
200	25.00	35.50	47.00

If  $\log P(f)$  is plotted as a function of  $\log f$ , the three curves are all straight lines, which is a typical character reflecting the fractal structure of  $P(f)$ . For example, both curves of specimen a (its  $\mu = 1556$  H/m and constant range is 0 ~ 240 A/m) and c (its  $\mu = 621$  H/m and constant range is 0 ~ 1194 A/m) are illustrated in figure 1. The curve of specimen b is similar to those of a and c.



**Figure 1  $\log P(f)$  vs.  $\log f$  curves of specimen a and c**

Figure 1 shows that when  $B_m = 0.1$  T the loss spectrum of three amorphous constant permeability alloys has fractal structure within the non-scale frequency range from 10 kHz to 200 kHz.  $P(f)$  can be unitedly given by the following fractal formula:

$$P(f) = P_0 f^{D_f} \quad (2)$$

where  $D_f$ , generally a fraction in value, is the fractal dimension of loss spectrum, and  $P_0$  is a parameter. The values of  $P_0$  and  $D_f$  are related to the intrinsic properties of materials and heat treatment technology conditions. The values of  $D_f$ ,  $P_0$ , anisotropy  $K_u$ , permeability  $\mu$  and magnetization field strength  $H$  are summarized in table 3.

The value of  $D_f$  is really a fraction (refer to table 3). Its meaning, describing the fractal structure of  $P(f)$ ,

**Table 3  $D_f$ ,  $P_0$ ,  $K_u$  and  $\mu$  of the specimens**

Specimen	$D_f$	$P_0 \times 10^6$	$K_u / \text{J} \cdot \text{m}^{-3}$	$\mu / \text{H} \cdot \text{m}^{-1}$	$H / \text{A} \cdot \text{m}^{-1}$
a	1.08	47.90	10.15	1556	0 ~ 240
b	1.25	8.56	42.00	840	0 ~ 796
c	1.27	9.16	65.00	621	0 ~ 119

differs from that of integral number 2 and 1 appearing in the traditional eddy-current loss  $P_e \propto f^2$  and magnetic hysteresis loss  $P_h \propto f^1$  formulas respectively.  $D_f$  indicates that the character—the total loss, caused by various mechanisms (such as eddy-current, magnetic hysteresis, relaxation, etc.), increases with the increasing frequency. The larger  $D_f$  is, the worse the frequency characteristic of  $P(f)$  is.

## 2.2 A Comparison between two $P(f)$ formulae

For specimen a, when  $B_m = 0.1$  T, the fractal formula of loss spectrum can be written as:

$$P(f) = 47.90 \times 10^{-6} f^{1.08} \quad (\text{W/kg}) \quad (3)$$

The traditional  $P(f)$  formula can be obtained with quite high confidence level according to formula (2), it is:

$$P(f) = 0.70 \times 10^{-10} f^2 + 1.14 \times 10^{-4} f - 0.17 \quad (\text{W/kg}) \quad (4)$$

Through equations (3) and (4), the values of  $P(f)$  in different frequencies can be calculated (listed in table 4).

**Table 4 A comparison between two formulae of  $P(f)$**

$f / \text{kHz}$	$P / \text{W} \cdot \text{kg}^{-1}$		
	observed	formula (3)	formula (4)
10	1.00	0.99	0.97
20	2.08	2.12	2.13
50	5.70	5.68	5.70
70	8.20	8.17	8.15
100	11.90	12.02	11.93
150	18.90	18.63	18.51
200	25.00	25.12	25.43

From table 4, it is found that the values of  $P(f)$  calculated using the two formulae agree with the observed values well. In traditional formula (4), the eddy-current loss ( $0.70 \times 10^{-10} f^2$ ) is low at the above frequency range. For example, when  $f = 100$  kHz, the eddy-current loss is only 6% of the magnetic hysteresis loss ( $1.14 \times 10^{-4} f$ ) and  $P_e / P_h = 12\%$  as  $f$  rises to 200 kHz. This result indicates the fact that the total power loss of amorphous alloy at high frequency is mainly caused by magnetic hysteresis. In fractal formula (3), this fact can be revealed by fractal dimension  $D_f$ . For specimen a,  $D_f = 1.08$  and is close to the integral number 1 appearing in traditional magnetic hysteresis loss  $P_h \propto f^1$ , which directly indicates that the

total power loss is mainly determined by magnetic hysteresis loss if  $D_f \rightarrow 1.0$ . The residual loss  $P_c$  in the tradition formula (4) is negative, which is difficult to explain in physics. While the fractal formula (3) has no such difficulty. As discussed above, though the two formulae can describe the behaviors of the total power loss spectrum, the fractal formula is better.

For specimen c, the fractal and traditional formulae of  $P(f)$  are

$$P(f) = 9.16 \times 10^{-6} f^{1.27} \quad (\text{W/kg}) \quad (5)$$

and

$$P(f) = 1.50 \times 10^{-10} f^2 + 21.58 \times 10^{-4} f - 2.14 \quad (\text{W/kg}) \quad (6)$$

respectively. There still exists a negative component ( $P_c = -2.14$  W/kg) in the traditional formula (6), though (6) closely corresponds to the observed  $P(f)$ . It can be calculated by formula (6) that when  $f = 100$  kHz,  $P_c/P_h = 7\%$ ; when  $f = 200$  kHz  $P_c/P_h = 14\%$ . Both values are larger than 6% and 12% of specimen a, so it is concluded that the increase of  $D_f$  means the increasing of the ratio of eddy-current loss to magnetic hysteresis loss.

As noted in this section, the fractal formula (2) can describe the behavior of core loss. The value of fractal dimension  $D_f$  can not only indicate the frequency characteristic of  $P(f)$ , but also represent the ratio of eddy-current loss to magnetic hysteresis loss qualitatively, thus reveals the main mechanism causing the total loss.

Through the transverse magnetic heat treatment, anisotropy is induced in amorphous constant permeability alloys. The procedure of magnetization is dominantly the rotation of magnetic domain. Though magnetic domain rotation may also result in the anomalous eddy-current loss [4], the value is quite low. Hence the core loss is mainly produced by magnetic hysteresis, which is consistent with the small values of  $D_f$  in the three alloys ( $D_f = 1.08, 1.25$  and  $1.27$ ).

### 2.3 Influence of induced anisotropy energy $K_u$ on $D_f$

Permeability  $\mu$  is related to the saturation magnetic induction  $B_s$  and induced anisotropy energy  $K_u$  ( $\mu \propto B_s^2/K_u$ ). After  $B_s$  is determined by selecting the chemical compositions of alloy, the increase of  $K_u$ , will result in the decrease of  $\mu$  and widen the constant range. Here it must be pointed out that  $K_u$  affects not only  $\mu$  directly but also fractal dimension  $D_f$  greatly.

The curves of  $D_f$ , vs  $K_u$  are shown in figure 2.

The decrease of  $K_u$  will cause the value of  $D_f$  to decrease (shown in figure 2), and thus, reduces the total power loss and improves the frequency character-

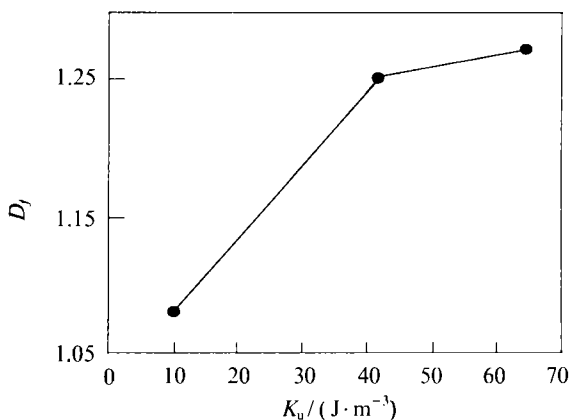


Figure 2 The dependence of  $D_f$  on  $K_u$  in amorphous constant permeability alloys

istic of  $P(f)$ , but at the same time, inevitably results in the increase of  $\mu$  and a narrow constant range. In order to develop the constant permeability alloy with exceedingly wide constant range, besides searching for the appropriate chemical compositions of alloy, We can only rely on the heat treatment conditions to obtain a large value of  $K_u$  so that the value of  $\mu$  can be decreased. However, the increase of  $K_u$ , in turn, causes  $D_f$  to rise and thus increases the total power loss and worsens the frequency characteristic. Therefore, it is difficult to develop the wide constant range alloy with extremely low power loss and excellent frequency characteristic.

### 3 Conclusions

(1) Within a high frequency range from 10 kHz to 200 kHz and at a constant  $B_m = 0.1$  T, the power loss spectrum  $P(f)$  of three kinds of Co-based amorphous soft magnetic constant permeability alloys has the fractal structure  $P(f) = P_0 f^{D_f}$ , where  $D_f$  is fractal dimension.

(2) In Co-based amorphous soft magnetic constant permeability alloys, with the increasing of induced anisotropy energy  $K_u$ , the fractal dimension  $D_f$  rises. Thus the total power loss at high frequency increases and the frequency characteristic of  $P(f)$  becomes worse.

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