Journal of University of Science and Technology Beijing
Vol.6(1999), No.3, p.157

Mineral

Modeling of Mechanical Behavior of Fractal Rock Joint

Jinan Wang[1], *Marek A. Kwasniewski*[2]

1) Resources Engineering School, University of Science and Technology Beijing, Beijing100083, China
2) Rock Mechanics Laboratory, Faculty of Mining and Geology, Technical University of Silesia, Gliwice, Poland
(Received 1998-112-09)

Abstract: The present study shows that naturally developed fracture surfaces in rocks display the properties of self-affine fractals. Surface roughness can be quantitatively characterized by fractal dimension D and the intercept A on the log-log plot of variance: the former describes the irregularity and the later is statistically analogues to the slopes of asperities. In order to confirm the effects of these fractal parameters on the properties and mechanical behavior of rock joints, which have been observed in experiments under both normal and shear loadings, a theoretic model of rock joint is proposed on the basis of contact mechanics. The shape of asperity at contact is assumed to have a sinusoidal form in its representative scale r_c with fractal dimension D and the intercept A. The model considers different local contact mechanisms, such as elastic deformation, frictional sliding and tensile fracture of the asperity. The empirical evolution law of surface damage developed in experiment is implemented into the model to up-date geometry of asperity in loading history. The effects of surface roughness characterized by D, A and r_c on normal and shear deformation of rock joint have been elaborated.

Key Words: fractal roughness; mechanical behavior; rock joints; fractal dimension; the intercept; contact mechanics; normal stiffness; shear stiffness; surface damage

1 Introduction

It has been well known that the existence of rock joints affects the deformation, strength and fluid conduction of rock masses considerably. Patton [1] studied the influence of various types of asperities on the friction resistance of joints and found that both asperity angle and asperity number would give rise to the change in failure envelope and, hence, the failure mode of joints. Barton and Choubey [2] proposed a sophisticated empirical model to predict the shear strength of rock joints by considering of normal stress, joint wall compressive strength and joint roughness coefficient (JRC). Since the roughness of joints plays such an important role, various joint models have been developed so far to constitute the property and mechanical behavior of rock joints by taking into account the effect of surface roughness. The models can be roughly divided into two groups: one is extrapolated on the global response of the joints under loading; the other is derived on the local contact mechanics [3–14]. The main idea of these approaches is that the global mechanical behavior of rock joint is resulted from the local contact mechanisms.

The important work involved, however, is to properly quantify the roughness. Statistical parameters as proposed in conventional approach and applied to describe surface roughness, such as the *r. m. s.* (root-mean-square) of asperity height, the *r.m.s.* of the first derivative of profile (the slope of asperity), the *r.m.s.* of the second derivative (the curvature of asperity), and so on, are not only quite complicated, but also suffer from scale effect which arises from sampling length, sampling interval and the resolution of the digitizing instrument. Consequently, they are not unique if different scale parameters are used, particularly for the type of rock fracture surfaces.

Investigations in recent years show that naturally developed fractures in rocks behave like fractals, and some pioneer work have employed fractal geometry [15] as an alternative method to characterize the surface roughness of rock joints [16–21]. Fractal geometry suggests that the structural phenomena exit at any scales. This scaling behavior (fractal in nature) can be quantitatively described by fractal dimension. A broad of study show, however, that most of rock fracture surfaces has the property of self-affinity rather than self-similarity. When fractal geometry is employed for the characterization of rock joint roughness, one or more scale parameters should be taken into account. To gain a better understanding of the structural behavior of fracture surfaces within the framework of fractal geometry, the fractal characters of fracture surfaces in rocks have been investigated in the present work. The effects of surface roughness characterized by fractal parameters on the mechanical behavior of rock joint are present on the theoretical basis of contact mechanics.

2 Fractal Characters of Rock Fracture Surfaces

By employing a non-contact optical instrument, a 3D laser profilometer, the fracture surfaces induced in rocks under different failure mechanisms were measured. The scanning results are analyzed following the variogram method [22]:

$$V(r)=\frac{1}{N-j}\sum_{i=1}^{N-j}[z(x_i+r)-z(x_i))]^2=Ar^{\beta} \qquad (1)$$

where, $V(r)$ is the variance of surface heights $z(x_i)$ parting from a distance of r, A is the intercept on V-axis of the log-log plot of variance *vs* r, and β is the slope of the plot. Fractal dimension is calculated by $D=2-\beta/2$ for a sectional profile. By comparing with the prescribed geometric profiles, two important parameters have been found to be related with surface roughness: fractal dimension D quantifies the irregularity of the rough surface while the intercept A is statistically analogous to the slope of asperity on the rough surface, *i.e.*, the more irregular of a rough surface, the higher the fractal dimension; and the steeper of the asperity, the greater the value of the intercept. For a rock fracture profile, however, the linear co-relationship between the variance and the distance on the log-log plot exists only in a limited range. The correlation length r_c statistically equals the half wave length of a periodic rough surface and, therefore, basically demarcates the structural elements—roughness and waviness—of the fracture surface in rock [23].

Fracture in rock usually extends in plane in which the crack front advances in a convex shape. In the process of crack propagation, the fracture mechanism may meander along different directions and in different places, resulting in both preferred and implicated cracking paths. The measurement shows that naturally developed fracture surfaces in rocks display the property of self-affine fractal. Fractal dimensions in adjacent profiles of 0.25 mm apart are different from each other and spatially distributed over the fracture surface. The values of fractal dimension and the intercept of a cross-sectional profile along the direction of fracture incitation appear to be lower than those of other fracture directions [24, 25]. The measurements also show that fractal dimension and the intercept suffer from the scale effect, which arises from the sampling length and sampling intervals. In general, the longer a sampling length and the smaller a sampling interval, a higher fractal dimension will be obtained. On the other hand, the longer a sampling length or the larger a sampling interval, the lower value of the intercept will be produced. Fractal parameters as applied to quantify the roughness of a fracture surface present the scale properties. Constant estimates on D and A could be made by the increasing of the sampling length and the decreasing of the sampling intervals to some extent.

The effectiveness of fractal roughness on the properties and mechanical behavior of rock joints have been investigated by means of triaxial compression and modified shear tests [26]. The joints under investigation include extension, shear and hybrid fractures induced in sandstones. In general, a rock joint with a high fractal dimension and a low value of the intercept will give rise to a high stiffness of normal and shear deformation. The shear strength of rock joints primarily depends on normal force F_n (kN). The influence of roughness on shear strength is of secondary importance, and in most cases, is not straight forward a single parameter dependent. Statistical analysis on the experiment results shows that the cross effect of fractal dimension D and the intercept A on the shear strength F_{tp} (kN) can be expressed by

$$F_{tp}=c_1\frac{V(r_c)}{\sqrt{A}}\sigma_T+c_2\log\left[\frac{A}{\sqrt{V(r_c)}}\right]\times F_n \qquad (2)$$

where σ_T is the tensile strength of the rock material, MPa; c_1, c_2 are regression constants with the values of 1.585 and 1.029, respectively. The variance at the maximum correlation length $V(r_c)$ here implies the scale effect.

During shear process, the degradation of surface roughness occurs. Based on the measurement of fracture surface before and after shear experiments, the surface damage law has been developed, in which the evolution of fractal roughness—D and A under shear can be empirically expressed by

$$D_S=D_0\left[1-c\frac{\sqrt{V(r_c)}}{\sigma_T}\log\left(\frac{\int \mathrm{d}W_P}{\sqrt{V(r_c)}}\right)\right] \qquad (3)$$

$$A_S=c_1'A_0+c_2'\sqrt{V(r_c)}\exp\left[-\frac{A_0\int \mathrm{d}W_P}{\sigma_T\sqrt{V(r_c)}}\right] \qquad (4)$$

where, D_S and A_S are the fractal dimension and the intercept after the amount of value of plastic shear work W_p (kN·mm) was done, $V(r_c)$ is the variance (cf. equation (1)) by taking the value at the correlation length r_c, and σ_T (MPa) is the tensile strength of the rocks; c, c_1', and c_2' in equations (3) and (4) are material constants [23, 26].

3 Modeling of the Effect of Fractal Roughness on Mechanical Behavior of Rock Joint

The extensive studies in experiments show that the mechanical behavior of rock joints is related to the surface structure, *i.e.* roughness. In fact, the global response to the loading of joints is controlled by the local contact mechanisms. Majumdar & Bhusha [27, 28] developed a fractal elasto-plastic contact model. It considers a flat plane in contact with a statistically isotropic rough surface at tip of asperities which is described by fractal dimension D and a characteristic length scale G. Based on Hertzian contact theory, the model shows that for elastic deformation, the normal load P and the real area of contact A_r are related as $P \sim A_r^{(3-D)/2}$; for plastic deformation, the load and contact area are linear related. The model predicts that the number of contact spots contributing to a certain fraction of the real area of contact remains independent of load although the spot size grows with load. Moreover, the load—area relation and the fraction of the real area of contact in elastic and plastic deformation are quite sensitive to the fractal parameters of surface roughness. Warren and Krajcinovic [29] developed a model of elastic-perfectly plastic contact of rough surfaces based on the Cantor set. They show that although the deformation of elasticity increases with fractal dimensions of rough surface under different levels of load, it is only a small part of the total deformation. The deformation of elasticity occurs mainly at a lower level of load for the surface with a lower value of fractal dimension. As fractal dimension increases, the deformation of plasticity remarkably increases and becomes the main part of the total deformation, particularly at a higher level of load.

These fractal contact models mentioned above account for normal contact behavior in absence of tangential force within the interface. For rock joints, however, the interlock of asperities is common and determines the shear behavior under a general loading condition. In the present study, a constitutive model of rock joint has been derived on the basis of fractal geometry and contact mechanics, in which some important contact mechanisms at asperities, such as elastic, frictional plastic deformation, and fracture of asperity, have been introduced into the model to show the influence of fractal parameters of surface roughness on the mechanical behavior of rock joint. Analytically, surface roughness is composed of numerously superimposing of asperities. Providing that a single asperity has a sinusoidal form, a single asperity is then expressed by a periodic function $z(x)$

$$z(x)=\sqrt{A}\,r^{(2-D)}\sin\frac{\pi x}{r} \qquad (0\leqslant x\leqslant r) \tag{5}$$

where D is fractal dimension, A is a coefficient which retains the same meaning in equation (1), and r_w is defined as the wave length of the representative asperity (**figure 1**). The radius of curvature is

$$R(x)=\frac{\left[1+\left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2\right]^{3/2}}{\left|\frac{\mathrm{d}^2z}{\mathrm{d}x^2}\right|}=\frac{\left[1+\pi^2Ar_w^{2(1-D)}\cos^2\frac{\pi x}{r_w}\right]^{3/2}}{\pi^2\sqrt{A}r_w^{-D}\sin\frac{\pi x}{r_w}} \tag{6}$$

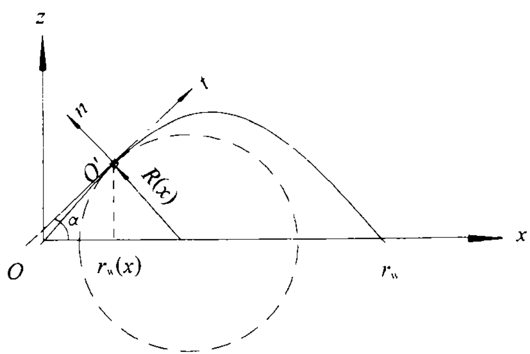

Figure 1 Schematic asperity

Suppose a joint has the same radius of curvature and the same elastic constant on either side, according to contact mechanics [30], the constitutive relationship at local contact asperities has been derived in an incremental form [23]:

$$\mathrm{d}P=\left\{\frac{3}{4}\frac{E^2P}{(1-\nu^2)^2}\frac{\left[1+\pi^2Ar^{2(1-D)}\cos^2(\pi x/r)\right]^{3/2}}{\pi^2\sqrt{A}r^{-D}\sin(\pi x/r)}\right\}^{1/3}\mathrm{d}\delta_n=[k_n]\,\mathrm{d}\delta_n \tag{7}$$

$$\mathrm{d}Q=\frac{2(1-\nu)}{2-\nu}k_n\left(1-\frac{Q}{\mu P}\right)^{1/3}\mathrm{d}\delta=[k_t]\,\mathrm{d}\delta \tag{8}$$

where P, Q are normal and tangential forces acting across the asperity at contact (**figure 2**), δ_n, δ_t are normal and shear displacements, k_n, k_t are coefficients of normal and tangential stiffness, E and ν are Young modulus and Posson ratio, and μ is the frictional fractor of the material, respectively. It is noticed that tangential stiffness in equation (8) depends not only on the surface roughness the same as the normal stiffness does, but

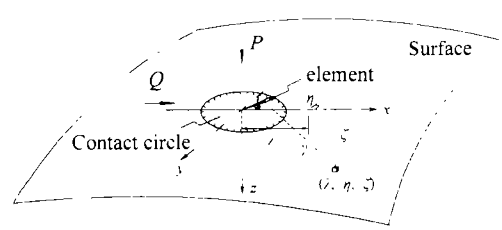

Figure 2 Stress analysis for a circle contact area

also on the ratio of tangential and normal forces applied. It is clear that when the tangential force exceeds the friction resistance at the contact point, sliding will take place along the asperity and a new contact configuration will be reached. The equilibrium state of contact requires assessment of force and displacement in reference to the current contact condition in successive incremental steps of load or displacement.

Under a sufficient high normal force, or when the friction within the interface is large enough to exclude net motion of the two surfaces, ($Q<\mu P$), brittle fracture might occur to the contact point. According to experimental observations [31, 32], fracture of asperities is possibly caused by tensile stress which is, therefore, of interest in the present study. According to contact mechanics, tensile stress σ_t induced by normal and tangential forces can be calculated from the distribution of forces within the contact area (figure 2) [23]:

$$\sigma_t=\sigma_t^p+\sigma_t^q \tag{9}$$

Under global system, the normal and shear forces applied to the joint (defined as F_n and F_t in **figure 3**) are

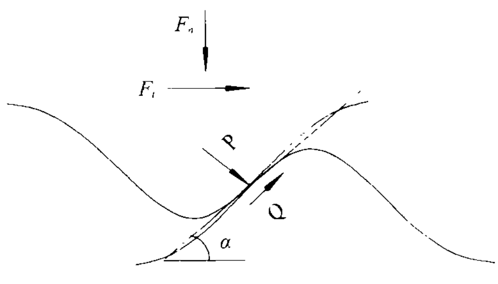

Figure 3 Transformation from local to global coordinate

transformed as follows:

$$\begin{Bmatrix} dF_n \\ dF_t \end{Bmatrix}=\begin{bmatrix} K_{nn} & K_{nt} \\ K_{tn} & K_{tt} \end{bmatrix}\begin{Bmatrix} du_n \\ du_t \end{Bmatrix} \tag{10}$$

The explicit expression of the contents in stiffness matrix above are given by

$$K_{nn}=k_n\cos^2\alpha+k_t\sin^2\alpha \tag{11a}$$

$$K_{nt}=K_{tn}=(k_n-k_t)\cos^2\alpha\sin\alpha \tag{11b}$$

$$K_{tt}=k_n\sin^2\alpha+k_t\cos^2\alpha \tag{11c}$$

where k_n and k_t are normal and tangential stiffness obtained on the basis of the local contact of asperities (cf. equations (7) and (8)). At any instant of time or sliding history, the angle of transformation is given by

$$\alpha=\tan^{-1}(\frac{dz}{dx})=\tan^{-1}(\pi\sqrt{A}r_w^{(1-D)}\cos\frac{\pi x}{r_w}) \tag{12}$$

The present model has derived from local contact mechanics, and therefore, can comprise different mechanisms of contact at asperities into a uniformed program. When slip or fracture occurs to asperity, for instance, the empirical surface damage law, which has been developed in experiments (cf. equations (3) and (4)) [26], is extended into the model to up-date the contact position as well as the shape of asperity.

4 Numerical Implementation

A number of examples are presented in this section to investigate the effects of surface roughness on the mechanical behavior of rock joints. Numerical implementation simulates the modified shear test, in which, initial normal and tangential load is applied to the asperity. Then, a prescribed displacement $u(\theta)$ is constantly applied obliquely to the joint plane. The basic parameters used in the modeling of rock joints are as follows: $E=30106$ MPa, $\nu=0.147$, $\sigma_T=8.0$ MPa, $\mu=0.3$, $\theta=40°$, and $\Delta u(\theta)=0.01$ mm.

In the modeling procedure, the contact conditions - elastic, plastic sliding and fracturing of asperity are automatically assessed through the following criteria:

(a) elastic contact: $Q<\mu P$ and $\sigma_t<\sigma_T$;

(b) sliding contact: $Q>\mu P$;

(c) fracture at contact: $\sigma_t>\sigma_T$.

A new contact configuration is up-dated according to the current state of contact. The plastic work is calculated in case the sliding occurs or the calculated tensile stress in the vicinity to the contact area exceeds the tensile strength of the material. The shape of asperity is then modified by recalculating fractal dimension D_s and the intercept A_s according to the empirical evolution laws of surface damage (equations (3) and (4)).

Numerical simulations reveal the significant effects of fractal parameters on the mechanical behavior of rock joint:

(1) Effects of fractal dimension (D). At a constant intercept ($A=1.0\times10^{-4}$), different values of fractal dimension are calculated ($D=1.1, 1.2, 1.3, 1.4$). As it is shown in **figure 4**, a joint with a higher fractal dimension gives rise to a high normal stiffness and higher shear stiffness. These results are consistent with experiments [23, 24, 26].

(2) Effects of the intercept (A). At a constant value of fractal dimension ($D=1.2$), four values of the intercept A are simulated ($A=1.0\times10^{-2}$, 1.0×10^{-3}, 1.0×10^{-4} and 1.0×10^{-5}). Numerical simulation shows that the smaller intercept (smooth surface) produces a higher normal stiffness of rock joint (**figure 5**). Since a bigger value of the intercept indicates a steeper slope of asperities [24, 25], the results highly cooperate with the

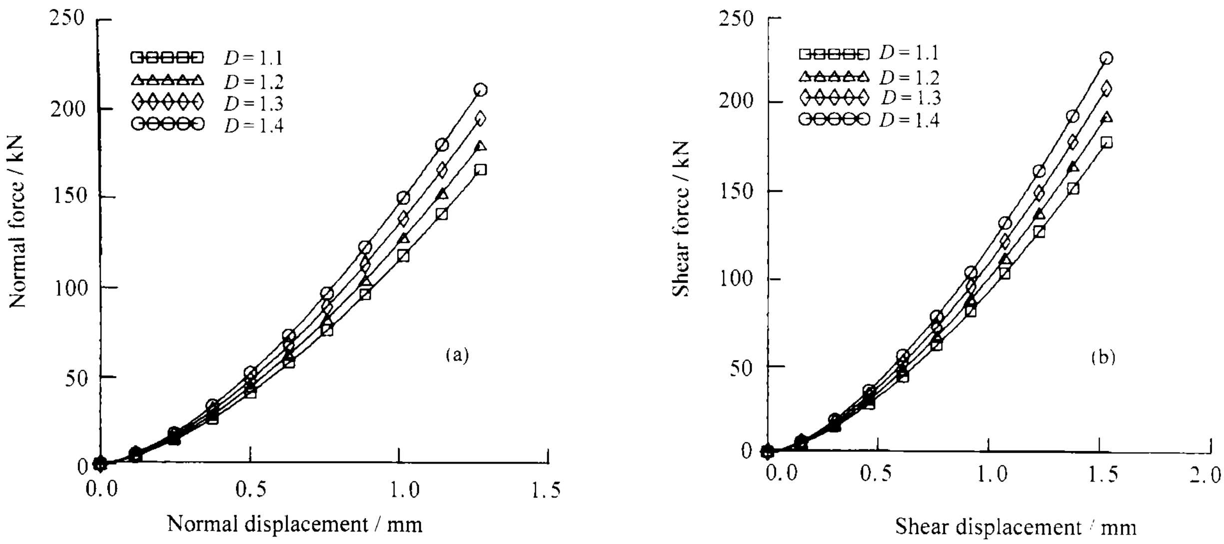

Figure 4 Effect of fractal dimmension D on (a) normal and (b) shear deformation of joint

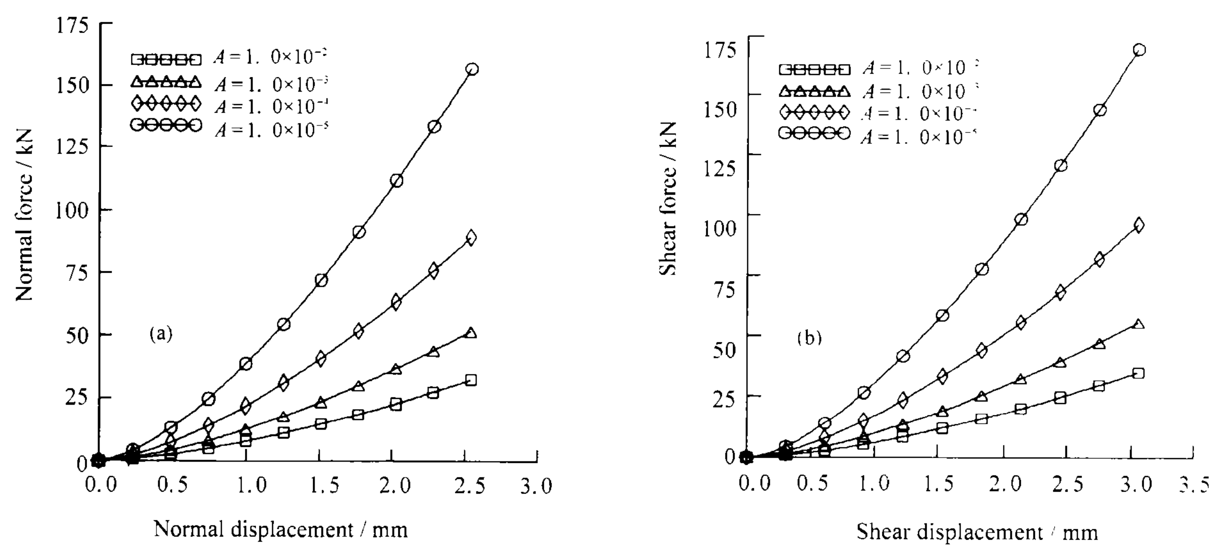

Figure 5 Effect of the intercept A on (a) normal and (b) shear deformation of joint

physical sense and agree well with experimental measurements. As shear stiffness is proportional to normal stiffness, a joint with a bigger value of the intercept gives rise to a lower shear stiffness, which has not yet been fully reasoned though it is convincingly supported by experiment evidence.

(3) Effects of the maximum correlation length (r_c). Equation (1) is stationary indicating that the mean squared increments of the rough surface depend only on the separating distance. A great number of measurements for real rock fracture surfaces show, however, that the linear relationship between the variance and the distance on the log-log plot exists only in a limited range, indicating the periodicity of the fracturing behavior. **Figure 6** shows the effect of r_c on the normal and shear deformation of rock joints under different values of r_c= 5, 10, 15 and 20 mm, respectively, where D = 1.2 and A =1.0×10^{-4}. As it is shown, the greater the r_c is, the greater the normal and shear stiffness will be.

(4) Effects of shear angle (θ). In modified shear tests, different shear angle will give various ratios of normal to shear force. The higher the shear angle θ, the higher the normal force F_n. To show the effect of shear angles, three angles are simulated from 30° to 40° and 50°, respectively, here fractal dimension D and the intercept A are chosen to be D =1.3, A=1.0×10^{-4}. As it is seen in **figure 7**, the shear angles has almost no influence on normal stiffness, only the ultimate normal displacements obtained at a higher shear angle. However, the effect of shear angles on shear stiffness is much pronounced, *i.e.* high angle of shear will produce a relatively high shear stiffness.

In addition to the effects of fractal parameters as pre-

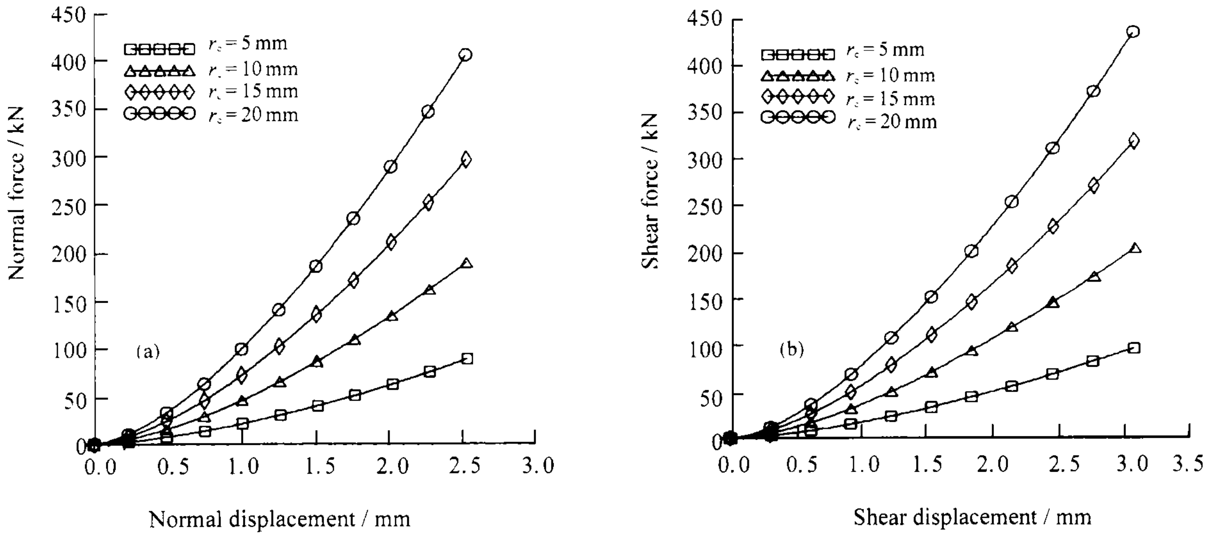

Figure 6 Effect of the correlation length r_c on (a) normal and (b) shear deformation of joint

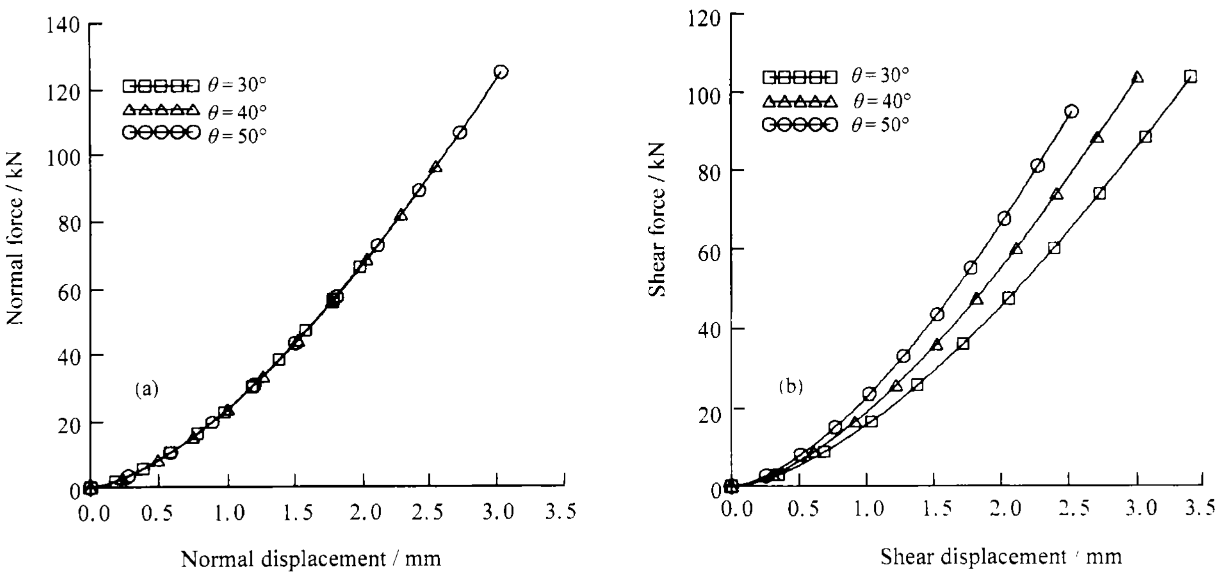

Figure 7 Effect of shear angle θ on (a) normal and (b) shear deformation of joint

sented above, a number of the actual shear tests of rock joints have also been simulated to show the performance of the model. The comparison of the predictions with the measurements is given in **figure 8**. It can be seen, at a low level of load, the predicted normal deformation satisfactorily agrees with the measurements; at a high level of load, however, the predicted normal deformation gradually deviates from the measurements. Since the present constitutive model has been derived based upon the contact geometry of a single pair of asperities, the interaction of neighboring asperities has been neglected. In fact, as increasing in load, some small asperities will emerge to form a large asperity and some new contact spot will be created [28, 33~35]. The present model could be improved by considering the contact size-distribution and the number of contact spots with respect to the load.

In the pre-failure part of shear, the shear deformations predicted by the model are in good agreement with experimental results. After peak shear strength, the competition of friction mechanism occurs between the joint surface and the boundaries of the experimental apparatus. The post-failure behavior of shear is, therefore, partially controlled by the boundary conditions. The modeling of post-failure behavior of rock joints might be more instructive if the boundary conditions are specified.

4 Conclusions

In view of the difficulties in characterization of surface roughness which prove to affect the properties and mechanical behavior of rock joints considerably, the fractal properties of fracture surfaces in rocks have

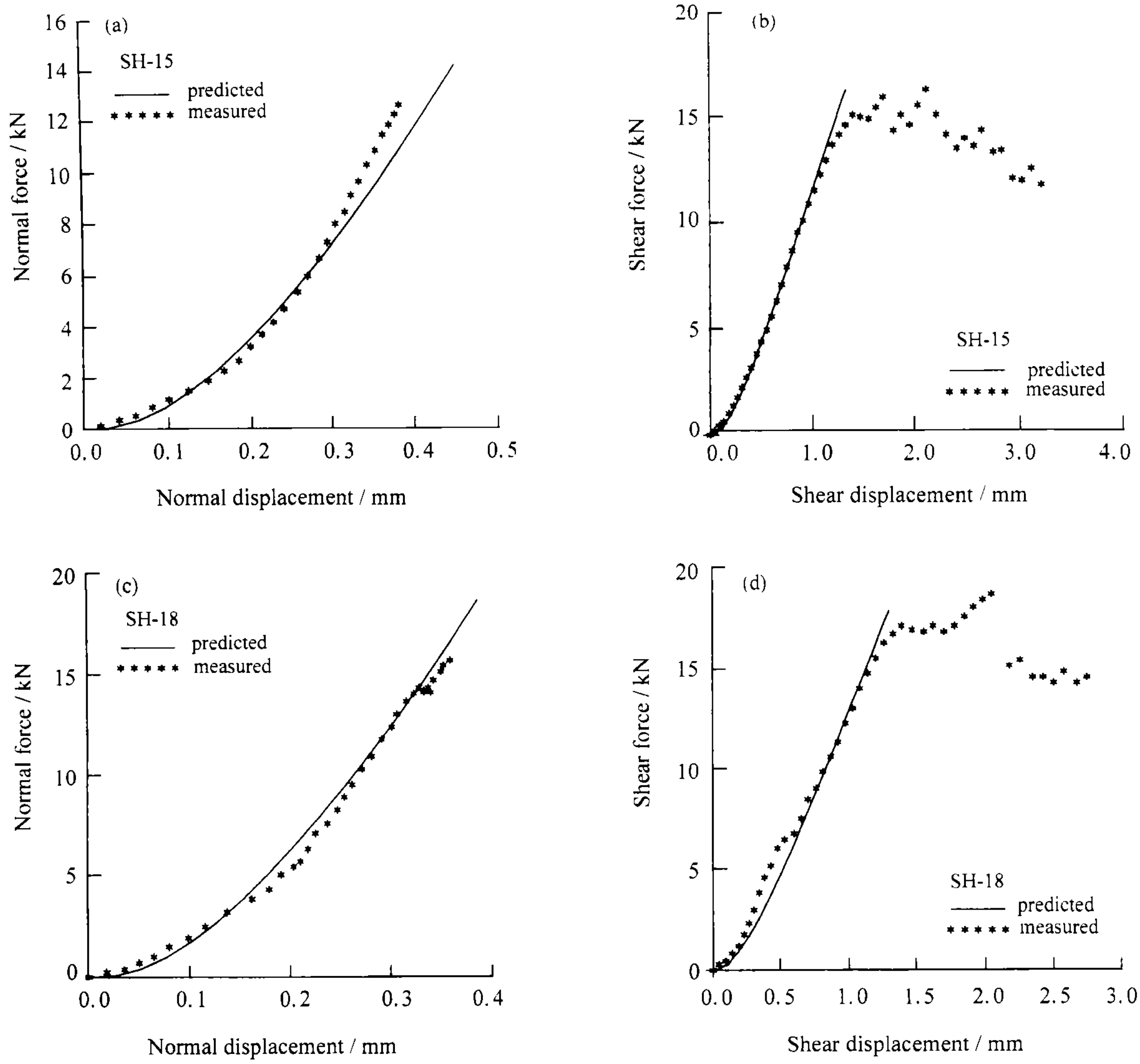

Figure 8 Modeling of modified shear tests (SH-15, SH-18), (a), (c)– normal deformation; (b), (d) – shear deformation.

been investigated. The analysis shows that naturally developed fracture surface behaves like self-affine fractals. The surface roughness can be quantitatively characterized by fractal dimension which describes the irregularity of the roughness, and the intercept of variance on its log-log plot which is closely related to the slope of asperities. In addition, the correlation length of the variance demarcates the roughness and waviness of a periodic fracture surface. Although fractal parameters may suffer from scale effect, a constant estimate could be expected by the increasing of the sampling length and the decreasing of the sampling intervals to some extent.

Experiments have shown the evidence of fractal parameters of surface roughness in influence of the properties of rock joints under shearing [25]. To confirm the significant findings, a constitutive model of rock joint is derived according to contact mechanics. Numerical investigation reveals that fractal dimension D, the intercept A and the correlation length r_c are three important parameters for the characterization of the surface roughness of rock joints. High values of D will give rise to lower normal stiffness and higher shear stiffness. On the contrary, the low value of A will produce a high normal stiffness and a low shear stiffness. Moreover, the greater the correlation length r_c is, the greater the normal and shear stiffness will be. These results are convincingly in agreement with the experimental measurements. The ratio of normal/tangential force has less effect on normal stiffness, while it results in different normal deformation regarding the magnitude of the normal force obtained. The shear stiffness appears to be normal force dependent. The higher the normal force, the higher the shear stiffness.

The simulations for the modified shear tests performed upon the constitutive model suggest that the modeling of both normal and shear deformation prior to the peak shear strength are in good agreement with observations. A satisfactory model of rock joint, however, could be derived by considering the effect of the increase in contact spots and the total contact area due to the emerge of neighboring asperities with load.

Acknowledgement

The present work has been financially supported by Post Doctoral Research Foundation of China Education Ministry and Polish-China Cooperation Project entitled ' Mechanics of Fractured and Jointed Rock Masses' under grant No.97/263.

References

[1] F. D. Patton: [In:] *Proc. 1st Cong. Int. Soc. Rock Mech.*, Lisbon, 1966, p.509.

[2] N. Barton, V. Choubey: *Rock Mechanics*, 10 (1977), No. 2, p. 1.

[3] J. A. Greenwood, J. B. P. Williamson: *Proc. Roy. Soc. London*, Ser. A 295 (1966), p. 300.

[4] P. R. Nayak: *J. Lubricat. Technol.*, 193 (1971), p. 398.

[5] A. W. Bush, R. D. Gibson, T. R. Thomas: *Wear*, 35 (1975), p. 87.

[6] A. W. Bush, R. D. Gibson, G. P. Keogh: *ASME*, Paper 78 (1978), LUB-16.

[7] M O'Callaghan, M. A. Cameron: *Wear*, 36 (1976), p. 79.

[8] G. Swan: *Rock Mech. and Rock Engng.*, 16 (1983), p. 16.

[9] Z. Sun: [In:] *Fundamentals of Rock Joints*, O. Stephansson [eds.:], CENTEK Publishers, 1985, p. 173.

[10] S. R. Brown, C. H. Scholz: *J. Geophys. Res.*, 90 (1985), p. 5531.

[11] M. E. Plesha: [In:] *Proc. 26th U.S. Symp. Rock Mech.*, 1985, p. 387.

[12] M. E. Plesha, B. C. Haimson: [In:] *Proc. 29th U.S. Symp. Rock Mech.*, 1988, p. 199.

[13] A. Zubelwicz, O. Connor, C. H. Dowding, T. Belytschko, M. E. Plesha: *Constitutive laws for Engineering Materials - Theory and Applications. II*, 1987, p. 1137.

[14] N. G. W. Cook: *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, 29 (1992), No. 3, p. 189.

[15] B. B. Mandelbrot: *The Fractal Geometry of Nature*. W.H. Freeman, New York, 1982, p. 468.

[16] S. R. Brown, C. H. Scholz: *J. Geophys. Res.*, 90 (1985), No. B14, p. 575.

[17] S. M. Miller, P. C. McWilliams, J. C. Kerkering: [In:] *Proc. 31th U.S. Symp.*, 1990, p. 471.

[18] Y. H. Lee, T. R. Carr, D. J. Barr, C. J. Hass: *Int. J. Rock Meck. Min Sci. & Geomech. Abstr.*, 27 (1990), p. 453.

[19] S. L. Huang, S. M. Oelfke, R. C. Speck: *Int J. Rock Mech. Min. Sci. & Geomech. Abstr.*, 29 (1992), No. 2, p. 89.

[20] H. Xie: *Fractals in Rock Mechanics*, Balkema, Rotterdam, 1993, p. 453.

[21] N. E. Odling: *Rock Mech. Rock Engng.*, 27 (1994), No. 3, p. 135.

[22] M. A. Kwasniewski, J. A. Wang: [In:] *Géotechnique et Environnement*, J. P. Piguet, F. Homand [eds.:], Vand uvre-lès-Nancy, Sciences de la Terre, 1993, p.163.

[23] J. A. Wang: *Morphology and Mechanical Behavior of Rock Joints [Ph.D. Thesis]*, Faculty of Mining and Geology, Silesian Technical University, Gliwice, Poland, 1994.

[24] M. A. Kwasniewski, J. A. Wang: *18th Winter School of Strata Mechanics*, Szkarska Poreba, Poland, 1995, p.71.

[25] M. A. Kwasniewski, J. A. Wang: *Int. J. Rock Mech. Min. Sci.*, 34 (1997), No. 3/4, p. 709.

[26] J. A. Wang, M. A. Kwasniewski: [In:] *Environmental and Safety in Underground Construction*, Lee, Yang & Chung [eds.:], Balkema, Rotterdam, 1997, p. 635.

[27] A. Majumdar, B. Bhushan: *J. Tribology Trans. ASME*, 122 (1990), p. 205.

[28] A. Majumdar, B. Bhushan: *J. Tribology Trans. ASME*, 123 (1991), p. 1.

[29] T. L. Warren, D. Krajcinovic: *Int J Solids Structures*, 32 (1995), No. 19, p. 2907.

[30] K. L. Johnson: *Contact Mechanics*, Cambridge University Press, 1985, p. 452.

[31] J. M. Handany, E. R. Danek, R. A. D. Andrea, J. D. Sage: [In:] *Rock Joints*, N. Barton, O. Stephansson [eds.:], Balkema, Rotterdam, 1990, p.195.

[32] Fishman, A. Yu: [In:] *Rock Joints*, N. Barton, O. Stephansson [eds.:], Balkema, Rotterdam, 1990, p. 627.

[33] A. J. Hyett, J. A. Hudson: [In:] *Rock Joints*, N. Barton & O. Stephansson [eds.:], Balkema, Rotterdam, 1990, p. 227.

[34] H. Xie, J. A. Wang, W. H. Xie: *Int. J. Rock Mech. Min. Sci.*, 34 (1998), No. 5, p. 865.

[35] F. Re, C. Scavia, A. Zaninetti: *Int. J Rock Mech. Min. Sci.*, 34 (1997), No. 3/4, p. 526.