Journal of University of Science and Technology Beijing
Vol.6(1999), No.3, p.165

A Quadratic Programming Model for Blast Scheduling

Chunyan Meng[1], *Samuel Frimpong*[2], *Mingjian Zuo*[3]

1) Ph. D. Student (correspondent), 606-Chemical-Mineral Building, Dept of Civil and Environmental Engineering, University of Alberta, T6G 2G6, Edmonton, AB. Canada 2) Associate professor, School of Mining and Petroleum Engineering, University of Alberta 3) Associate professor, Department of Mechanical Engineering, University of Alberta

(Received 1999-02-05)

Abstract: A quadratic programming model is established to choose the blocks to be blasted in a given period. The length of this period depends on the production planning requirements. During the given period, the blocks' parameters are available from the geological database of the mine. The objective is to minimize the deviation of the average ore grade of blasted blocks from the standard ore grade required by the mill. Transportation ability constraint, production quantity demand constraint, minimum safety bench constraint, block size constraint and block, bench precedence constraints are considered in forming the programming model. This model has more practical objective function and reasonable constraints compared with the existing model for this kind of problems.

Key words: quadratic programming; open pit mining; blast scheduling; mine production scheduling; mathematical programming model

1 Introduction

In the area of open pit mining, the sequence of ore and waste extraction has become even more important and given the trend towards larger pits and mining equipment. Open pit mining involves a continuous change of the open pit surface with many feasible plans of extraction leading to the same ultimate pit limits. Each of these plans generates a production schedule over the entire mine life, which may or may not be optimal. The development of an optimal production sequence in open pit mines is a very complex procedure, due to the very large number of variables and constraints involved. It is this complexity, however, that creates an ideal environment for the application of the sophisticated operations research and computer techniques. There are quite a few references, so far, of applications of linear of integer programming in solving the open pit scheduling problem.

However, very little information is given in these references on the practicality of the mining schedules produced [1]. One of the common problems with such highly sophisticated mathematical programming algorithms is that they are literally 'black box' not allowing any interference by the model user in order to set his own priorities and adapt the system to the real case study. Some of these sophisticated algorithms may produce mining schedules which are not at all practical in terms of mining practice [1].

The production planning deals mainly with the problems of: (1) selecting the blocks to be blasted in the benches exposed for mining; (2) deciding the quantities of ores and wastes to be excavated in each time period from each of the blasted blocks; and blending of the mined ores so that the periodwise demands, in terms of quantity and quality, for ores can be met utilizing the limited resources available to the mine.

In the production planning, selecting the blocks to be blasted in a given period, *i.e.*, blast scheduling, is also a key procedure. Because the following excavating procedure is based on the already blasted blocks in a short term [1], the optimal blast scheduling with practical considerations is very important to the optimum production planning. This makes blast planning and the production planning problem inseparable. But, in reality, the blast scheduling problem is seldom solved through the mathematical programming methods. The literature research by Ei Compendex and GeoRef (1982–1998) show that only one paper [2] reported the study on blast scheduling using mathematical programming method.

The blast scheduling is chosen for this paper. The production planning is a very large problem. It will be introduced in another paper. In this project, a specific nonlinear programming method—quadratic programming method will be developed to handle the blast scheduling problem with several practical considerations.

2 Preparation for Blast Scheduling Model

2.1 Related geological data and block model

In this section, some foundations are given, such as how to form the blocks, the basic features of the block, what data is available for the optimal problem.

The creation of a geological model is necessary to characterize a resource in terms of its shape, size, quality or grade, quantity or tonnage, variability and its economic and geological limits, using data acquired from the ore body and results of the analysis and interpretation of that data.

Most geological model are developed in 3-D and may be simple or complex depending on the nature of the resource, the data available and the degree of sophistication in the studies made of the resource. The model should contain all the data obtained about the deposit including structural, chemical, mineralogical and physical [3].

Models should attempt to explain all the observable facts. Data should not be ignored if it does not fit the model. The reasons for the misfit must be found. Most geological modeling is now computerized with all the relevant data retained in databases. Computer operations are based on three activities—input, processing and output. The main task of the engineer is to ensure that the input data is correct and relevant; that the correct process for manipulating the data is selected; and that the correct results are produced and are recognized as being correct.

The block model is the basis for almost all computer supported pit designs. For a block model, a rectangular block large enough to cover the area of interest is placed around the mineral deposit. The large block is then subdivided into smaller three-D blocks. The smaller blocks may be of various different sizes and shapes.

The geometrical position of a block is uniquely fixed with reference to any suitable coordinate system. Each block is assigned geological, rock mechanical, processing and economic data pertaining to each type of material contained in the block. There are various types of block models: the regular 3-D fixed block model is the most widely used type in practice. The vertical height of each block is usually the bench height and horizontal shape will normally be a rectangle or a square. The main characteristics of a 3-D fixed block model are that each block is of the same measurements.

The assignment of data to each block is effected by various interpolation techniques. Three such techniques are: Geostatistics using Krigging; inverse distance weighting methods; and the method of polygons. In a physical sense the block model will take the form of a computer data file.

2.2 Model justification

Usually the blasting occurs once a week. The blocks available depend on the drilling conditions. Each set of drill is arranged in a working zone. To form the optimization problem, we should know which blocks could be blasted. This can be got according to the production plan. Each block's quality can be got from the corresponding mine geological database.

The short term blast schedule is seldom subject to the block precedence or bench precedence constraint, because there are usually enough exposed blocks for blasting. Furthermore, this can be avoided when setting the drills.

For the purpose of modeling, the ore body has been divided into several benches of 12 m height and each bench into rectangular blocks of 30 m×30 m size. A smaller size could not be used as the accuracy of the kriging estimators becomes questionable if the block size is less than 30 m×30 m for a bore hole grid size of 60 m×60 m. A block in the ore body is uniquely defined by specifying the column numbers (grid numbers) in *x*, *y*, *z* directions from the origin (**figure 1**).

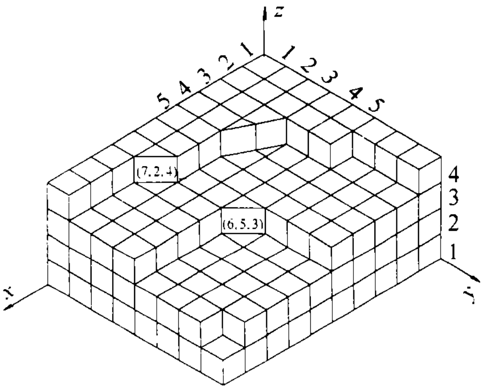

Figure 1 The regular 3-D fixed block model of deposit

3 Mathematical Model of the Optimal Blast Scheduling

The objective function will minimize the deviation of average ore grade blasted during a given period from the ore grade required by the mill. The function will look like equation (1).

Minimize $D=$

$$\left\{\frac{\sum_x\sum_y\sum_z Q_{\text{oldo}x,y,z}\cdot K_{\text{Fe}x,y,z}+O_{x,y,z}\cdot K_{\text{Fe}x,y,z}\cdot P_{x,y,z}}{\sum_x\sum_y\sum_z Q_{\text{oldo}\,x,y,z}+O_{x,y,z}\cdot P_{x,y,z}}-K_{\text{required}}\right\}^2 \quad (1)$$

Constraints:

(1) Transportation constraint:

$$\left\{\sum_x\sum_y\sum_z\left[(Q_{\text{oldo}\,x,y,z}\cdot TrO_{x,y,z}+Q_{\text{oldw}x,y,z}\cdot TrW_{x,y,z}+(O_{x,y,z}\cdot\right.\right.$$

$$TrO_{x,y,z}+W_{x,y,z}\cdot TrW_{x,y,z})\cdot P_{x,y,z}]\Big\}\Big/Cap \le ToTr \qquad (2)$$

(2) Production quantity requirement:

$$\sum_x\sum_y\sum_z(Q_{\text{oldo}.x,y,z}+O_{x,y,z}\cdot P_{x,y,z}) \ge De \qquad (3)$$

(3) Safety bench constraint:

$$(Y_{\max.z}-y)\cdot P_{x,y,z} \ge Sw \qquad \text{for all } x, y \text{ and } z. \qquad (4)$$

Given the available blocks, we know that every variable is named accordingly. Then we can write the constraint in details.

(3) Block size constraint:

$0 \le \mathrm{P}_{x,y,z} \le 1$ for all x, y and z.

Notation:

$P_{x,y,z}$: A variable that takes value one if block $B_{x,y,z}$ is to be fully blasted and zero if it is not to be blasted. Fractional values of $P_{x,y,z}$ represent the portion of block to be blasted;

$B_{x,y,z}$: A block located in the intersection of xth, yth and zth columns;

$O_{x,y,z}$: The quantity of unblasted ore in block $B_{x,y,z}$;

$W_{x,y,z}$: The quantity of unblasted waste in block $B_{x,y,z}$;

$\mathrm{Q}_{\text{oldo}.x,y,z}$: The quantity of uncleared ore of the previous blasts in block $B_{x,y,z}$;

$Q_{\text{oldw}.x,y,z}$: The quantity of uncleared waste of the previous blasts in block $B_{x,y,z}$;

$TrO_{x,y,z}$: Average transport cycle time for ore from block $B_{x,y,z}$ to crushing mill;

$TrW_{x,y,z}$: Average transport cycle time for waste from block $B_{x,y,z}$ to dumping site;

$ToTr$: Total transportation time available;

$K_{\text{Fe}\,x,y,z}$: Average quality of mineral Fe in block $B_{x,y,z}$, *i.e.* ore grade;

K_{required}: The iron ore grade required by the mill;

Sw: Safety width (minimum width to be maintained for any bench in terms of block width);

$Y_{\max.z}$: The value of y where this block holds the wall of the pit;

Cap: Capacity of the dumper;

De: Ore quantity demand.

4 Solution method of the optimal problem

The observation of this problem: all constraints are linear; objective function is the quotient of two quadratic forms. The original problem is equivalent to minimize the following quadratic function subject to the same constraints as before:

$$D_2=\Big\{\sum_x\sum_y\sum_z(Q_{\text{oldo}\,x,y,z}\cdot K_{\text{Fe}\,x,y,z}+O_{x,y,z}\cdot K_{\text{Fe}\,x,y,z}\cdot P_{x,y,z}-$$

$$K_{\text{required}}\cdot\Big(\sum_x\sum_y\sum_z(Q_{\text{oldo}\,x,y,z}+O_{x,y,z}\cdot P_{x,y,z})\Big)\Big\}^2 \qquad (5)$$

The procedure of solving the above problem is:

(1) Introduce slack and redundant variable to the corresponding constraint respectively and form equality constraints. Then get a quadratic programming problem in the standard form of

Minimize $f(x)=cx+x^{\mathrm{T}}\mathrm{Q}x$,

s.t. $Ax=b$,

$x \ge 0$.

(2) Write out the Kuhn-Tucker conditions

$$c-x^{\mathrm{T}}(Q+Q^{\mathrm{T}})-u-v=0$$

$$Ax=b,\ x\ge 0,$$

$$Ux=0,\ u\ge 0.$$

(3) Solve the K-T conditions and get the optimal solution of the original problem. The solution may be obtained by using the modified Linear Programming simplex method or the complementary pivot method [4].

5 Discussion and Conclusions

Under these constraints, a specific non-linear optimization problem is formed, *i.e.* a quadratic programming problem. The problem described here has one more practical consideration, which lies in: minimizing the grade deviation from the required grade, not maximizing the number of the blasted blocks which was introduced by D. K. Sundar [2]. There should no more difficulty in solving the quadratic problem using the modified LP simplex method than solving the LP problem formed by the above author. However, considering only the next given period of blast scheduling is apparently only the first step in the production planning process. Besides being a very helpful tool for this purpose the model will have much more efficiency when utilized as part of a greater planning system for medium and long range scheduling.

Reference

[1] K. Fytas, J. Hadjigeorgiou, J. L.Collins: *International journal of surface mining and reclamation*, 7 (1993), No. 1, p. 1.

[2] Diatha Krishna Sundar, Damodar Acharya: *Computer Industry and Engineering*, 28 (1995), No. 4, p. 927.

[3] W. Hustrulid, M. Kuchta: *Open pit mine planning and design: fundamentals*, Vol.1. A. A. Balkema, Rotterdam, The Netherlands, 1995.

[4] G. V. Reklaitis, A. Ravindran, K. M. Ragsdell: *Engineering Optimization: Methods and Applications*. John Wiley, Toronto, 1983.