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Materials

Genetic Algorithm for the Thermal Stresses Optimum Design of Functionally Gradient Material Plate

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Abstract: Based on the thermal stress distribution for functionally gradient material (FGM) plates, a Genetic Algorithm (GA) method for the thermal stresses optimum design of FGM plate with computer technologies is given. The minimum thermal stresses combination distribution for FGM is obtained.

Key Words: functionally gradient material (FGM); thermal stress; Genetic Algorithm (GA); crossover; mutation

Genetic Algorithms are exploratory search and optimization procedures that are devised on the principles of natural evolution and population genetics. One of the attractions for researchers in GA as a search and optimization technique was that it could obtain near- optimal solutions to different types of problems without requiring optimized object being derivative or continuous, by simply manipulating bit strings. It has been large reported that GA were applied to function optimization over real R^n object spaces. However, the report that GA for the thermal stresses optimism design of FGM plate has not been seen.

FGM is a new kind of heterogeneous composite material that consists of a gradient compositional variation from ceramic to metal and from one surface to the other. It is a key problem to reduce thermal stress made from different effective material properties of intermediate compositions of the FGM and prevent destruction by thermal stress. In this paper, using GA method, optimum parameters matching are determined, which make the thermal stress of FGM plate minimum.

1 Analytical Model

Ceramic -metal FGM plate of thickness h is modeled as an asymmetric laminate of n layers which could represent a gradient of material distribution. The z -axis is taken vertical to FGM plate and x–y plane and middle plane of FGM plate coincide with each other as indicated in **figure 1**.

It is also assumed that the temperatures of upper and lower surfaces of the model are steadily maintained to T_U and T_L, respectively. According to T. Hirano [1], the volume ratio for the metal of FGM is given:

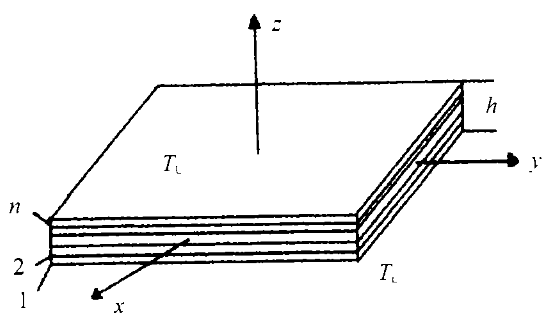

Figure 1 Analytical model

$$f(\xi)=\begin{cases} f_0 & 0\le\xi\le\xi_1 \\ (f_n-f_0)\left(\dfrac{\xi-\xi_1}{\xi_{n-1}-\xi_1}\right)^p+f_0 & \xi_1\le\xi\le\xi_{n-1} \\ f_n & \xi_{n-1}\le\xi\le 1 \end{cases} \tag{1}$$

where $\xi_1=\dfrac{t_a}{h}$; $\xi_{n-1}=\dfrac{h-t_b}{h}$; $\xi_0=0$; $\xi_n=1$; t_a, t_b are the thickness of ceramic and the thickness of metal in FGM plate, respectively; and by equation(1), ξ_i ($i=2,\cdots,n-2$) is obtained:

$$\xi_i=\left(\frac{f_i-f_0}{f_n-f_0}\right)^{1/p}(\xi_{n-1}-\xi_1)+\xi_1 \tag{2}$$

where f_i is the volume ratio of metal in the ith layer which is given in advance. The temperature distribution and thermal stress distribution are developed as follows.

(1) The temperature distribution

$$T(z)=T_L+\frac{(T_U-T_L)}{\sum\limits_{i=1}^{n}\dfrac{h_i}{\lambda_i}}\left(\sum_{i=1}^{s}\frac{h_i}{\lambda_i}+\frac{z-z_s}{\lambda_{s-1}}\right) \qquad z_0\le z\le z_s \tag{3}$$

where $z_s=\xi_s h-\dfrac{h}{2}$; $h_i=z_i-z_{i-1}$; $s=0, 1, \cdots, n$; $i=1, 2, \cdots, n$; λ_i is a coefficient of heat conduction at the i-th layer, let $\sum_{i=1}^{0}=0$.

(2) The thermal stress distribution at i-th layer :

$$\sigma_{x,i}=\sigma_{y,i}=\sigma_i(z)=\frac{E_i}{1-\nu_i}(\varepsilon_0+zk_0-\alpha_i T(z)) \tag{4}$$

where E_i, ν_i, and α_i are Young modulus, Poisson ratio and the coefficient of thermal expansion at the i-th layer, respectively; ε_0, k_0 are the strain at the midplane and the curvature at the midplane, respectively, they are determined as follows:

$$\begin{bmatrix}\varepsilon_0\\ k_0\end{bmatrix}=\begin{bmatrix}A & B\\ B & D\end{bmatrix}^{-1}\begin{bmatrix}C\\ F\end{bmatrix},$$

constants A, B, C, D, F are calculated as follows [1]:

$$A=\sum_{i=1}^{n}\frac{E_i}{1-\nu_i}h_i,$$

$$B=\frac{1}{2}\sum_{i=1}^{n}\frac{E_i}{1-\nu_i}(z_i^2-z_{i-1}^2),$$

$$C=\frac{1}{2}\sum_{i=1}^{n}\frac{\alpha_i h_i E_i}{1-\nu_i}(T_{i-1}+T_i),$$

$$D=\frac{1}{3}\sum_{i=1}^{n}\frac{E_i}{1-\nu_i}(z_i^3-z_{i-1}^3),$$

$$F=\sum_{i=1}^{n}\frac{\alpha_i E_i h_i^2}{6(1-\nu_i)}\left(\frac{3z_i}{h_i}(T_{i-1}+T_i)-T_i-2T_{i-1}\right),$$

where $T_i=T(z_i)$.

The biggest thermal stress of FGM plate $\max\limits_{1\le i\le n}\max\limits_{z}\sigma_i(z)$ will be determined, while n, p, h, t_a, t_b are given. Therefore, $\max\limits_{1\le i\le n}\max\limits_{z}\sigma_i(z)$ is a function of n, p, h, t_a, t_b:

$$\sigma(n,p,h,t_a,t_b)=\max_{1\le i\le n}\max_{z}\sigma_i(z) \tag{5}$$

The optimization strategy in the design process is making the biggest thermal stress of FGM plate $\sigma(n,p,h,t_a,t_b)$ minimum by properly matching the number of layers n, the shape factor p, the thickness of FGM plate h and t_a, t_b.

2 An Overview of GA

It is difficult to minimize $\sigma(n, p, h, t_a, t_b)$ by traditional method, because the objective function can't be derived. The GA doesn't need derivative or other information of objective function.

As has been stated, GA imitates natural evolution, and hence includes operations such as "reproduction", "crossover", and "mutation". For a given population of individuals and set of operators together with procedures for evaluating each individual, a simple GA proceeds as follows:

1) Determining the coding of the parameters to be optimized;

2) An initial random population of trials: $P(0)=\{x(i, 0), i=1, 2, \cdots, M\}$, where M is the number of trials in the population, is generated;

3) The fitness of each trial, $F(x(i, k))$, $x(i, k)\in P(k)$, $i=1, 2, \cdots, M$, is calculated, where k is the number of generations of population, in initial generation, $k=0$;

4) One or more trials are selected by taking $x(i, k)\in P(k)$, using the probability distribution:

$$P_i^k=\frac{F(x(i,k))}{\sum_{j=1}^{M}F(x(j,k))}\quad (i=1,2,\cdots,M) \tag{6}$$

5) The genetic operators: crossover and mutation are applied to the selected trials in order to produce the next generation of population, $P(k+1)$, where the crossover operator is applied with probability p_c, and the mutation operator is applied in probability p_m.

6) The GA process is terminated after a prespecified number of generation or on the basis of a criterion which determines the convergence of the population.

3 GA for Optimum Design of FGM Plate

Consider:

$\min\sigma(n,p,h,t_a,t_b)$,

subject to : $1\le n\le N$, $p_-\le p\le p_+$, $0<h\le h_1$, $0<t_a\le t_1$, $0<t_b\le t_2$.

The optimum design procedure for FGM of GA is shown in **figure 2**.

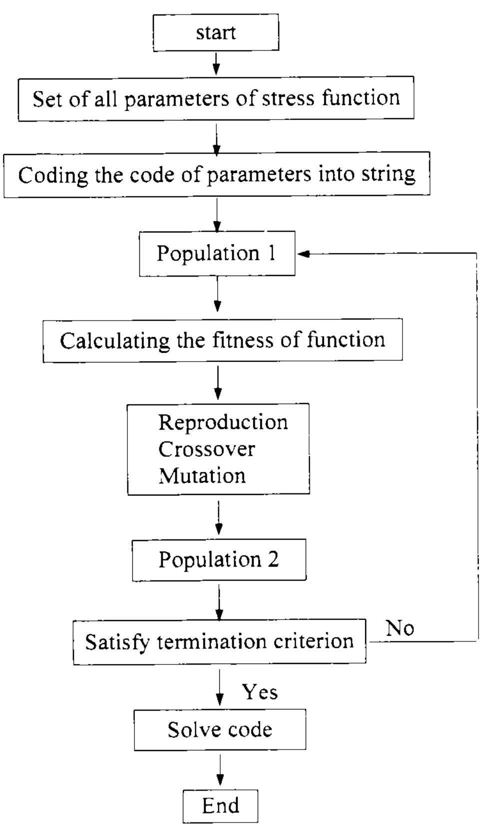

Figure 2 Design procedure of GA for FGM

3.1 Coding and creating an initial population

An initial random population of trials is taken as $P(0)=\{x(i, 0), i=1, 2, \cdots, M\}$, where $x(i, 0)$ is an individual, which is defined by $X(i, \bullet)=\{n, p, h, t_a, t_b\}$, M is the number of individuals in population. When the usual binary alphabet is used, the code of n, p, h, t_a, t_b is a l_i-tuple string u_i of 1s and 0 s, respectively. That is:

n: $u_1 = b_{11}b_{12}\cdots b_{1l_1} \in [0, 2^{l_1}]$,

$n = 1 + \delta_1 u_1$,

$\delta_1 = \frac{N-1}{2^{l_1}}$;

P: $u_2 = b_{21}b_{22}\cdots b_{2l_2} \in [0, 2^{l_2}]$,

$P = P_- + \delta_2 u_2$,

$\delta_2 = \frac{P_+ - P_-}{2^{l_2}}$;

h: $u_3 = b_{31}b_{32}\cdots b_{3l_3} \in [0, 2^{l_3}]$,

$h = \delta_3 u_3$,

$\delta_3 = \frac{h_1}{2^{l_3}}$;

t_a: $u_4 = b_{41}b_{42}\cdots b_{4l_4} \in [0, 2^{l_4}]$,

$t_a = \delta_4 u_4$,

$\delta_4 = \frac{t_1}{2^{l_4}}$;

t_b: $u_5 = b_{51}b_{52}\cdots b_{5l_5} \in [0, 2^{l_5}]$,

$t_b = \delta_5 u_5$,

$\delta_5 = \frac{t_2}{2^{l_5}}$,

where $b_i \in \{0, 1\}$.

The each individual structure using concatenated binary mapping is given by

$$x(i, \cdot) = \{u_1, u_2, u_3, u_4, u_5\} = \{b_{11}b_{12}\cdots b_{1l_1}, b_{21}b_{22}\cdots b_{2l_2}, b_{31}b_{32}\cdots b_{3l_3}, b_{41}b_{42}\cdots b_{4l_4}, b_{51}b_{52}\cdots b_{5l_5}\}.$$

The string length of an individual is

$$l = \sum_{i=1}^{5} l_i.$$

3.2 Objective function mapping onto fitness

Because the fitness function is always non- negative, and it always has been hoped that the better the higher of fitness values. The fitness function in this problem is taken as

$$F(x) = \begin{cases} G_{max} - \sigma(n, p, h, t_a, t_b), & \sigma < G_{max} \\ 0 & \text{others} \end{cases} \tag{7}$$

where the number G_{max} is observed maximum fitness value, $\sigma = \sigma(n, p, h, t_a, t_b)$ is defined by equation (5).

3.3 Propagation and termination

The fitness of each individual in a population $F(x(i, k))$, $(i=0, 1, \cdots, M)$ is evaluated. A new generation of population is formed by randomly selecting string from an existing population according to the probability distribution P_i^k. Then the crossover operator and the mutation operator are applied respectively with probability p_c and p_m. The new population is produced. The GA process is terminated when the criterion

$$|F(P(T)) - F(P(T+1))| < \varepsilon$$

or a prespecified number of generation is satisfied. The individual with the highest fitness is restored as decimal number, which is near optimal solution.

4 Experimental for TiB-Cu FGM

In this experiment, the thermal conditions are assumed that $T_L = 300$ K, $T_U = 1500$ K. The variables boundary values are taken as $t_a = t_b \in [0.5, 2.5]$, $h \in [3, 23]$, $n \in [2, 18]$, $p \in [0.2, 1.8]$, respectively. M (the size of population), p_c, and p_m are 216, 0.6 and 0.005, respectively. The length of the code l_i ($1 \leqslant i \leqslant 5$) is respectively 4, 4, 5, 5, 5. After GA process is performed 20 generations, the convergence criterion of the population is satisfied, and the near optimal solution is obtained:

$n = 13$, $p = 0.6$, $h = 9$ mm, $t_a = t_b = 0.5$ mm.

The minimum thermal stress is 169.6 MPa.

In order to verify the near optimal solution, let one variable vary, the distribution of thermal stress of FGM plate is shown in **figures 3, 4, 5**, respectively.

The minimum of thermal stress shown in the graphs is consistent with the calculating result.

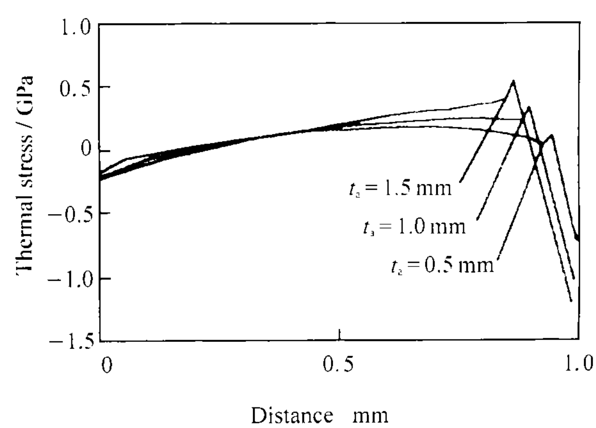

Figure 3 Influence of t_a on thermal stress

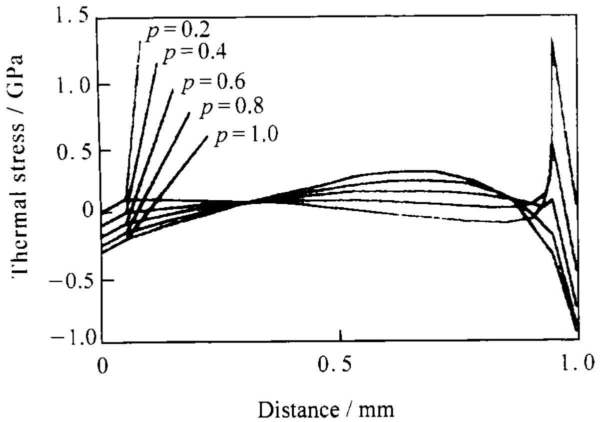

Figure 4 Influence of the shape factor p on thermal stress

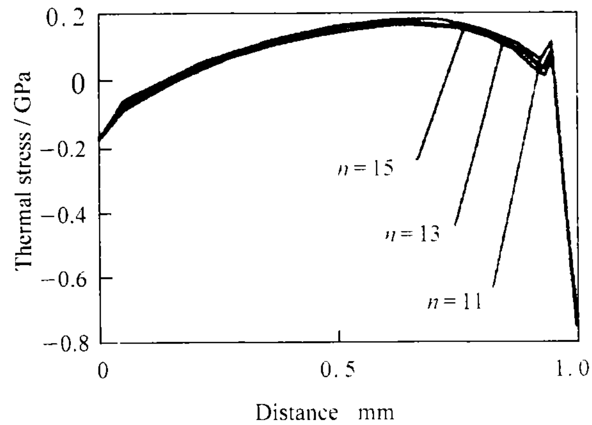

Figure 5 Influence of the number of layers n on thermal stress

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