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H^{∞} Theory and its Application to the Position of Rolling Mill Control System

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Abstract: The controller designed according to classical or modern control theory will not satisfy the performance requirements when the controlled object in industrial field can not be described by exact mathematical model or the disturbance of the controlled system. In order to make the controlled system stable and having good performance, H^{∞} control theory was put forward to solve this practical problem. Taking the position of a rolling mill as the controlled object, it was rectified by optimal engineering way. Then, three different plans were put forward according to Bang-Bang control, LQ control and H^{∞} control, respectively. The result of the simulation shows that the controller designed according to H^{∞} method whose robust performance and ability to restrain colors disturbance is satisfactory.

Key Words: position control; H^{∞} control; LQ control; Bang-Bang controll

1 Position Control System and Control Algorithms

The position of rolling mill control system belongs to APC(Automatic Position Control) system. The system makes the roller stop at the scheduled position by regulating the speed of the electromotors and a series of mechanical transmitted equipment. During transmission, there is friction between the screw and the shaft. The friction where the shaft usually works is small and the friction at both ends of the shaft is bigger. At the same time, in the transmitted equipment clearance is different at the different position, which is nonlinear and immeasurable disturbance of the system. The inertia torque of the system is different at different position of the roller, which can be thought that there is disturbance in the model of the object, and colors disturbance is introduced into the speed loop.

When the input signal belongs to some set or there is uncertainty in the object itself, how to design a robust controller? In order to make up the limitation of the modern control theory, robust control theory is developed. During the process of design, uncertain error of the mathematical model is considered, at the same time the error of the frequency performance or the error of the parameters between model and plant is allowed. H^{∞} control theory that developed in the recent twenty years is successful and perfect to solve robust problem [1]. In 1981, Canadian scholar Zames used H^{∞} form of the sensitivity function of the system to scale the performance when the system restrains some disturbance set [2]. Zames developed the H^{∞} algorithm of linear system. Later on, H^{∞} control theory about MIMO system and other nonlinear system were developed.

Model matching problem, minimum sensitivity problem and robust stability problem are often discussed in H^{∞} theory, these problems can be transformed to each other. Approximation theory, interpolation theory or ARE (Algebraic Ricatti Equation) solution are mainly used to solve these problems.

2 Model Establishment and Solution of Control Principle

The position of the rolling mill control system was taken as the plant, engineering optimal algorithm was used to revise the controlled object according to theoretical analysis, and then the system model was transformed into state equation. The position noted as x_1 and the speed noted as x_2 were chosen as state variables, u is input signal and position is output, the matrix equation can be described as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & \dfrac{1}{261}\\ 0 & -24.7\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 2470\end{bmatrix}u,$$

$$y=[0\ \ 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix}.$$

Different schemes according to the theories of Bang-Bang control, LQ control and H^{∞} control were put forward and simulated. In Bang-Bang control, the switch time T_1, T_2 must be solved. In LQ control, $u(t)$ must be calculated to minimize the performance

$$J(u)=\frac{1}{2}e^{T}(T_f)Fe(T_f)+\frac{1}{2}\int_{t_0}^{T_f}[e^{T}(t)Q(t)e(t)+u^{T}(t)R(t)u(t)]\mathrm{d}t$$

in order to make the output track to the given position quickly, where F and $Q(t)$ are positive semi-definite matrices. The optimal control principle is

$$u(t)=R^{-1}(t)B^{T}(t)[g(t)-K(t)x(t)],$$

where $K(t)$ is the solution of the Ricatti matrix differential equation

$$\dot{K}(t)=-K(t)A(t)-A^{T}(t)K(t)+K(t)B(t)R^{-1}(t)B^{T}(t)K(t)-C^{T}(t)Q(t)C(t) \quad (1)$$

which satisfies the limit condition

$$K(T_f)=C^{T}(T_f)FC(T_f)\,.$$

Vector $g(t)$ is the solution of the linear vector differential equation

$$\dot{g}(t)=-[A(t)-B(t)R^{-1}(t)B^{T}(t)K(t)]^{T}g(t)-C^{T}(t)Q(t)Z(t) \quad (2)$$

which satisfies the limit condition

$$g(T_f)=C^{T}(T_f)F\,Z(T_f).$$

In order to obtain the optimal control principle, 4-order Longue-Kutta was used to solve the equation. Then simulation was processed according to the principle.

In H^{∞} algorithm, two free degree control was used to solve the model matching problem. In many systems, the anticipate output characteristic can be obtained by the transfer function from the reference-input to the output of the plant, that is, for the reference-input $r(t)$, the close-loop system was designed to make

$$y=G_{yr}(s)r \quad (3)$$

where $G_{yr}(s)$ is the given ideal output model.

Single feedback control commonly can not solve this problem. Two free degree control method was put forward to add the free parameters during the process of design and the method can satisfy equation (3). **Figure 1** is a two free degree control system. It is composed of feedback controller C_2, feed-forward controller C_1 and plant P. Apparently the feed-forward C_1 cannot improve the stability of the close loop of the system. So the feedback controller C_2 must be designed to make the feedback system (which consists of P and C_2) stable and, if necessary, to make the whole system robust and stable.

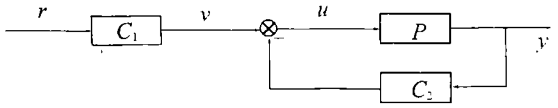

Figure 1 Block diagram of two free degree control system

Assuming that for the given $C_2(s)$, the transfer function from v to y is $G_{yv}(s)$, that is

$$G_{yv}(s)=P(s)[I+C_2(s)P(s)]^{-1},$$

let $C_1(s)=G_{yv}^{-1}(s)\,G_{yr}(s)$, then the close loop transfer function from r to y can satisfy equation (3). From the engineer angle, the feed-forward controller C_1 defined as above must satisfy the realizable condition: (a) $C_1(s)$ is analytical at close right s-plane; (b) $C_1(s)$ is a real rational function matrix.

For a given plant, assume that the anticipant output y_m of the controlled object is described as

$$y_m=G_m(s)r,$$

where r is the reference-input and $G_m(s)$ is the reference model. The controller

$$u=C_1(s)r+C_2(s)y \quad (4)$$

will be discussed, that is, the controller is composed of feed-forward and feedback.

Model matching problem was defined as follows. For a given controlled object $P(s)$ and a reference model $G_m(s)$, a controller (4) is designed to make the close loop transfer function $G_{yr}(s)$ from r to y equal to $G_m(s)$, so the output y equals to y_m for any reference input r.

The plant discussed in this paper was $y=G_{yu}(s)\,u$, where $G_{yu}(s)=\frac{9.46}{s(s+24.7)}=\frac{N(s)}{T(s)}$. To make the system arrive at the anticipant output quickly, the anticipant transfer function was set as

$$G_m(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}=\frac{1}{(0.02s+1)(0.03s+1)}=\frac{1665}{s^2+83.3s+1665}=\frac{N_d(s)}{T_d(s)}\,.$$

So the control principle is $u=C_1r+C_2y$. There is a 0-order polynomial $K(s)=k$ and 1-order polynomial $H(s)=s+a$, that satisfy

$$kT(s)+H(s)N(s)=Q_1(s)[T(s)-\frac{1}{9.46}Q_2(s)N(s)].$$

When data were substituted, we obtained

$$k(s^2+24.7s)+9.46\times H(s)=Q_1(s)[s^2+24.7s-Q_2(s)] \quad (5)$$

It could be assumed that 1-order polynomial $Q_1(s)=s+b$ and 2-order polynomial $Q_2(s)=(s+c)(s+d)$ whose 2-order coefficient was 1, where $b, c, d>0$. Inserted $Q_1(s)$ and $Q_2(s)$ into the equation (5) and rewrote it, we obtained

$$ks^2+(24.7k+9.46)s+9.46a=$$

$$(24.7-c-d)s^2+[b(24.7-c-d)-cd]s-bcd.$$

Comparing the coefficient and letting $c=1$ and $d=1$, it is obtained that $a=-2.66$, $b=25.16$, $k=22.7$, so $K(s)=22.7$, $H(s)=s-2.66$, $Q_1(s)=s+25.16$, $Q_2(s)=s^2+2s+1$. Therefore,

$$u=\frac{s-2.66}{s+2.46}y+\frac{(s+25.16)(s^2+2s+1)}{9.46(s+2.46)}\cdot\frac{1\,665}{s^2+83.3s+1\,665}r,$$

$$C_1(s)=\frac{(s+25.16)(s^2+2s+1)}{9.46(s+2.46)}\cdot\frac{1\,665}{s^2+83.3s+1\,665}=\frac{176(s^3+27.1s^2+51.32s+25.16)}{s^3+85.76s^2+1\,869.9s+4\,095.9},$$

$$C_2(s)=\frac{s-2.66}{s+2.46}.$$

If $C_1(s)$ and $C_2(s)$ are achieved, u can be obtained. The position can be controlled.

In order to transform C_1 and C_2 from frequency domain to time domain, the method below was adopted.

Taking $C_1(s)$ and $C_2(s)$ as transfer functions (here for example for $C_1(s)$), then

$$C_1(s)=\frac{y(s)}{u(s)} \tag{6}$$

Let $u(s)=\frac{1}{s}(u(t)=1)$, then $y(t)=C_1(t)\times1=C_1(t)$. If $y(t)$ is obtained, $C_1(t)$ also can be obtained.

Rewrite equation (6) as state equation

$$\begin{cases}\dot{x}_1=x_2-10\,313.6u\\ \dot{x}_2=x_3+5.64\times10^5u\\ \dot{x}_3=-4\,095.9x_1-1\,869.9x_2-85.76x_3-2.98\times10^7u\\ y=x_1+176u\end{cases} \tag{7}$$

Let the initial value $x_{10}=0$, $x_{20}=0$, $x_{30}=0$. If we take a proper step h and solve equation (7) by means of 4-order Longue-Kutta, $x_{1,j-1}$, $x_{2,j-1}$, $x_{3,j-1}$ can be obtained and if they are inserted into y, y_{j-1} can be obtained, which equals to $C_{1,j-1}$.

For $C_2(s)$, $\begin{cases}\dot{x}_1=-2.46x_1-5.12u\\ y=x_1+u\end{cases}$, if y_{j-1} is solved, $C_{2,j+1}$ can be obtained.

After solving C_1 and C_2, the optimal control principle $u(t)=C_1(t)s_g+C_2(t)x_2(t)$ can be obtained, where s_g is the given position and $x_2(t)$ is the speed state variable. According to that equation, simulation can be processed.

3 Simulation

In order to verify the validity and superiority of H^∞ designed algorithm, Gauss white noise and sine disturbance of the same class as well as the perturbation of the model's parameters were introduced into the systems which designed by different methods in the paper. Some simulation curves are shown in **figures 2–6**. From the simulation curves and data, it can be concluded as follows.

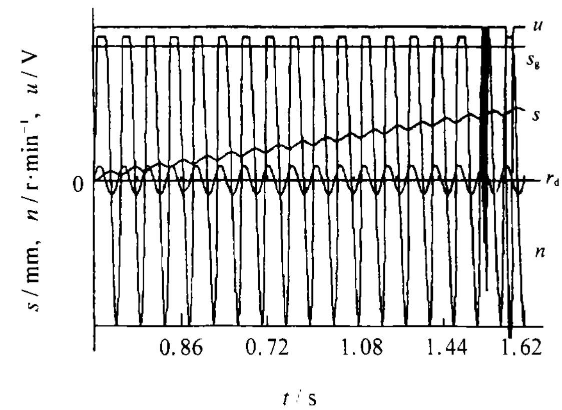

Figure 2 LQ controller (infinite time, sine disturbance (100 r / min)), s_g– the given position; s– the real position; r_d– disturbance (so as the follows).

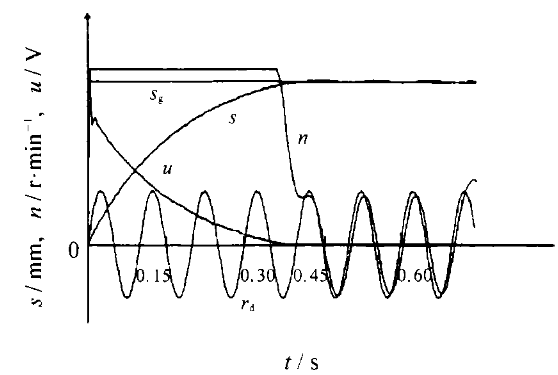

Figure 3 LQ controller (finite time, sine disturbance (100 r / min))

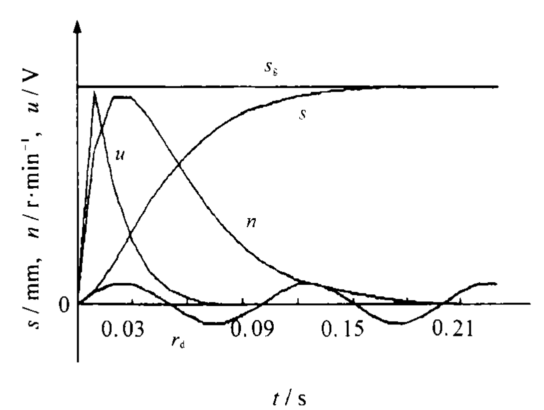

Figure 4 H^∞ controller (sine disturbance (100 r / min))

For Bang-Bang controller, the performance of the system about speed that has an exact model is perfect, but it can not restrain any disturbance. For LQ control-

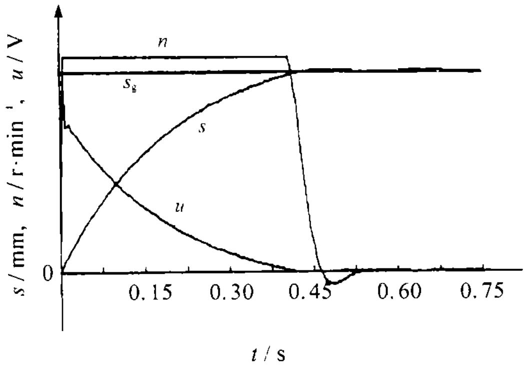

Figure 5 LQ controller (finite time, model's parameters perturbation (1.1))

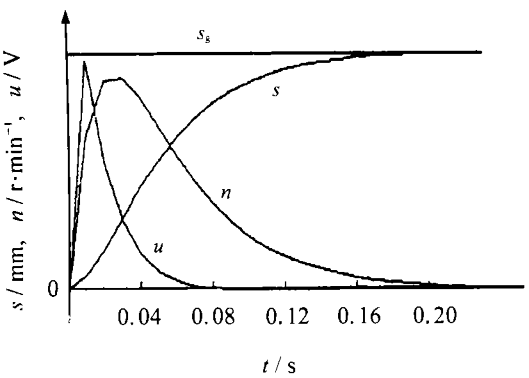

Figure 6 H$^\infty$ controller (model's parameters perturbation (1.1))

ler, the system can restrain white noise, and sine disturbance whose amplitude is small, for example 20 r/min, has little influence on LQ controller. However, the LQ controller can not restrain the sine disturbance whose amplitude is greater, for example 100 r/min, and the system may be unstable. When the model's parameters are perturbed, though LQ controller may have no influence on the utmost output, it delays the control period. For example, the time constant of the model is 1.1 times that of before, the control period varies from 0.5 s to 0.75 s.

The simulation result shows that the controller that designed according to H$^\infty$algorithm can restrain any disturbance even the parameters of the model varied in some range and the effect is better.

4. Conclusions

The controller designed basing on H$^\infty$ theory takes long computer time to transform the parameters from frequency domain to time domain or vice versa. In order to calculate H$^\infty$ controller, an engineer must grasp relatively abstruse mathematical theory. So it is necessary for us to make great effort to apply H$^\infty$ algorithm whose theory is precise and whose ability of solving problem is the greatest to the practice. Here we only had a try. We hope that the algorithm can be improved progressively.

References

[1] B. A. Francis: *A Course in H$^\infty$ Control Theory*, Spring-Verlag, New York,1987.

[2] G. Zames, B. A. Francis: *IEEE Trans. On Auto. Cont.*, AC-28 (1983), No. 5, p. 585.

[3] B. A. Francis, J. William Helton, G. Zames: *IEEE Trans. On Auto. Cont.*, AC-29 (1984), No. 10, p. 888.

[4] J. Dayle, D. Glover, D. Khargonekar, B. Francis: *IEEE Trans. On Auto. Cont.*, AC-34 (1989), p. 831.

[5] T. L. Shen: *H$^\infty$ Control Theory and its Application*, Tsinghua University Press, Beijing, 1996.