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A Mathematical Model for Predicting Shrinkage Defect of Ductile Iron Castings

Zhiqiang Han [1], *Junyi Su* [2], *Kaike Cai* [1]

1) Metallurgy School, University of Science and Technology Beijing, Beijing 100083, China
2) Xi'an Jiaotong University, Xi'an 710049, China
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Abstract: The shrinkage defect of a ductile iron casting is attributed to the volume variations occurring in solidification, which consist of liquid contraction, solidification shrinkage, graphitization expansion, and mold cavity enlargement. Based on this understanding, a mathematical model for predicting the shrinkage defect of the casting is developed, in which the volume variations of the casting in solidification are numerically simulated, especially, the mold cavity enlargement is quantitatively calculated. Moreover, the reliability of the model is verified in production and experiment.

Key words: ductile iron casting; shrinkage defect; mold cavity enlargement; mathematical model

Ductile iron is extensively used in machinery manufacturing industry because of its high strength, excellent castability, and inexpensive cost. However, the shrinkage defect is an outstanding problem in the production of ductile iron castings.

As it is known, numerical simulation plays an increasingly important role in analyzing solidification process, predicting shrinkage defect, and optimizing foundry technique. For a steel casting, the criterion for shrinkage defect prediction, $G/\sqrt{R}$, proposed by E. Niyama [1] is widely used, where G and R are respectively temperature gradient and cooling rate of the casting at the end of solidification. It is proved that this criterion is successful for steel casting, but not effective for ductile iron casting [2].

J. F. Wallace *et al.* [3] investigated the factors influencing the shrinkage defect such as mold condition, pouring temperature, liquid iron composition, and casting shape or modulus. From their results, it can be induced that the defect of the casting is attributed to the volume variations occurring in the solidification, which consist of liquid contraction, solidification shrinkage, graphitization expansion, and mold cavity enlargement. Therefore, directly simulating the volume variation occurring in solidification process is efficient to predict the defect of the casting.

In a previous paper [4], numerical simulation on the volume variations has been used to predict the shrinkage defect of a ductile iron casting, but the calculation on mold cavity enlargement was avoided by using dry sand mold in the study. J. F. Wallace *et al.* [3] showed that the mold cavity enlargement in later stage of solidification, which can not be fed by liquid iron coming from riser and compensated by graphitization expansion, is the main cause of the defect for the casting produced in green sand mold. The objective of this paper is to present a complete model for predicting the shrinkage defect of ductile iron castings, in which the volume variations including liquid contraction, solidification shrinkage, graphitization expansion, and mold cavity enlargement are numerically simulated.

1 Model Description

1.1 Calculation of temperature field

The temperature field in the casting is the basis of volume variation calculation and shrinkage defect prediction. So, firstly, the calculation of temperature field is introduced.

The heat transfer and solidification of the casting are assumed governed by the following 3-dimensional transient heat conduction equation,

$$\frac{\partial(\rho cT)}{\partial t}=\frac{\partial}{\partial x}(\lambda\frac{\partial T}{\partial x})+\frac{\partial}{\partial y}(\lambda\frac{\partial T}{\partial y})+\frac{\partial}{\partial z}(\lambda\frac{\partial T}{\partial z})+\rho G \quad (1)$$

where T is temperature, t is time, and ρ, c, λ are density, specific heat, and heat conductivity respectively. G is the rate of latent heat release. If one assumes that the release of the latent heat is proportional to the increase of the solid fraction, f_s, then G can be expressed by

$$G=L\frac{\partial f_s}{\partial t} \quad (2)$$

where L is the latent heat released during solidification.

In the model, the following relationship [5] between solid fraction and temperature is used,

$$f_S = 1 - e^{-4.005T^*} \quad (3)$$

where T^* is a dimensionless temperature and is expressed by

$$T^* = \frac{T_L - T}{T_L - T_S} \quad (4)$$

where T_L and T_S are the liquidus temperature and solidus temperature. For the commonly used ductile iron with eutectic composition, T_L and T_S represent the beginning temperature and the end temperature of the eutectic transformation.

By solving the above heat conduction equation, the temperature field in the casting is calculated and the distribution of solid fraction can be obtained by using equation (3).

1.2 Calculation of volume variation

After the temperature field and the distribution of solid fraction in the casting are obtained, the volume variations occurring in the solidification must be calculated for the purpose of shrinkage defects prediction. In solidification process of the casting, the volume variations such as liquid contraction, solidification shrinkage, and mold cavity enlargement occurring before the feeding channel is blocked are easily fed by the liquid iron coming from riser and/or sprue and will not lead to the shrinkage defect in the casting. But those occurring after the feeding channel is blocked, if they can not be compensated by the graphitization expansion completely, will exactly result in the defect in castings. In short, only the volume variations occurring after the feeding channel is blocked are directly related to the formation of shrinkage defect. Obviously, the major task of shrinkage defect prediction is to calculate the volume variations occurring after the feeding channel is blocked.

In present model, V_C is used to represent the total amount of liquid contraction, solidification shrinkage, and graphitization expansion occurring after the feeding channel is blocked, and V_m is used to represent the amount of mold cavity enlargement occurring after the feeding channel is blocked. The defect is predicted by calculating and comparing V_C and V_m. If V_C is larger than V_m, it can be predicted that the shrinkage defect does not appear. Otherwise, the defect is formed.

In the model, whether the feeding channel is blocked is judged by a critical solid fraction. If the average value of solid fraction in the feeding channel is larger than the critical solid fraction, the feeding channel is considered blocked.

(1) Calculation of V_C.

In computation, once the feeding channel is blocked, the temperature and solid fraction of each mesh element of the casting are saved immediately. The total amount of liquid contraction, solidification shrinkage, and graphitization expansion in the casting can be calculated by using these data. Firstly, the volume variation of one of the mesh elements is considered. In order to simplify the calculation, the solidification shrinkage and the graphitization expansion are combined as the volume variation during eutectic transformation for ductile iron with eutectic composition. If it is assumed that the liquid contraction is proportional to the temperature drop of liquid iron and the volume variation in eutectic transformation is proportional to the increase of solid fraction, the volume variation of the element occurring after the feeding channel is blocked can be expressed as follows,

$$V_C(i) = \begin{cases} [(\frac{T(i) - T_L}{100} \times r_L) + r_E] \times V(i), & T(i) \geq T_L \\ [1 - f_S(i)] \times r_E \times V(i), & T(i) \leq T_L \end{cases} \quad (5)$$

where $T(i)$, $f_s(i)$, $V(i)$ are the temperature, solid fraction, and volume of the element respectively. r_L is the volume contraction rate of the liquid iron for 100 ℃ temperature drop. r_E is the volume variation rate during eutectic transformation of the iron.

The total amount of liquid contraction, solidification shrinkage, and graphitization expansion in the casting can be obtained by accumulating the volume variation of each mesh element of the casting,

$$V_C = \Sigma V_C(i) \quad (6)$$

(2) Calculation of V_m.

Before introducing the method for calculating the mold cavity enlargement occurring after the feeding channel is blocked, it is necessary to review the understanding about the mechanism of the mold cavity enlargement and the relationship between the enlargement and the shrinkage defect. As it is known, in solidification of a ductile iron casting, the casting usually contacts with the mold perfectly [6], so, in this paper, the mold cavity enlargement and the swelling of the outer shell of the casting have the same mean.

J. F. Wallace *et al.* [7] measured the mold wall movement of green sand and no-bake molds in the solidification process of a cylinder ductile iron casting 50 mm in diameter and 250 mm in length. In the meantime, the temperature variation in the casting was also measured. The results are shown in **figure 1**. In this figure, the liquidus temperature T_L and eutectic temperature T_E were taken from a cooling curve at a point with 12 mm

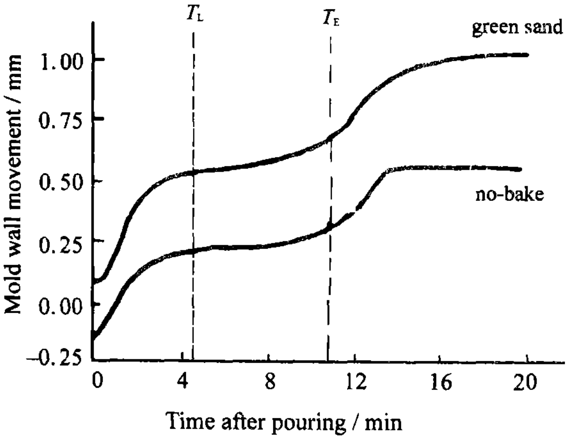

Figure 1 Mold wall movement of a cylinder ductile iron casting measured by J. F. Wallace [7], T_L is liquidus temperature; T_E eutectic temperature.

distance from the surface of the casting. It is found that the mold wall movement can be divided into two stages. The first stage is from the time at which the pouring is just finished to the time at which the temperature of the casting is down to liquidus. The second stage is from the beginning of eutectic transformation to the end of solidification. It was indicated in reference [3] that the movement during the first stage is caused by ferrostatic pressure. Generally, this movement can be fed by liquid iron coming from riser and/or sprue. However, the movement during the second stage o ften can not be fed by liquid iron due to the blocking of feeding channel, and leads to the formation of shrinkage defect.

E. Niyama *et al.* [8] also measured the mold wall movement and temperature variation of a cylinder ductile iron casting 50 mm in diameter and 120 mm in length. Furthermore, the relationship between the time after pouring and the solid fraction in the surface layer of the casting was determined by quenching the solidifying casting. **Figure 2** shows the plots of the movement of mold wall and the temperature in casting versus time. It is also found from the figure that the movement can be clearly divided into two stages. Moreover, the solid fraction in the surface layer of the casting is about 0.7 at the beginning of the second stage. In reference [8], the mechanism on the swelling of outer shell is described as follows. In solidification process, eutectic cells grow continuously. When the cells are in close contact with each other in the surface layer of casting, a solid skeleton entrapping residual liquid is formed there. Thereafter, the growth of eutectic cells will cause the swelling of the skeleton and mold cav ity enlargement. When the arrangement of the spheroids with the same size is under the most compact condition, the volume fraction of solid is about 74%. However, the eutectic cells are randomly arranged in liquid iron. It can be reasonably considered that the cells are in close contact with each other and the solid skeleton begins to be formed when the solid fraction is increased to 0.7.

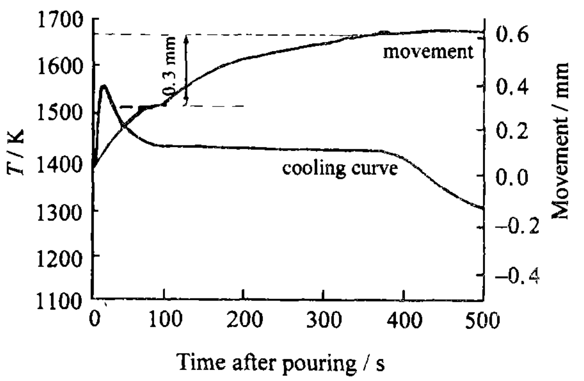

Figure 2 Mold wall movement of a cylinder ductile iron casting measured by E. Niyama [8]

Based on the above understanding on the mechanism of mold cavity enlargement and the experiment results, the following method for calculating the enlargement occurring after the blocking of feeding channel is presented.

1) Determining the total amount of mold cavity enlargement occurring in whole solidification process. The total amount of mold cavity enlargement is influenced by mold condition, liquid iron composition, shape and size of casting *etc*. To date, it can not be quantitatively modeled yet. In this study, for the practical purpose, the total amount of mold cavity enlargement occurring in whole solidification process of a real casting is obtained by comparing the dimensions of the casting with those of the pattern and considering the solid contraction after solidification.

2) Dividing the total amount of mold cavity enlargement into two parts. In light of the basic opinion on the mechanism of mold cavity enlargement and the experiment results in references [3, 8], the total amount of mold cavity enlargement is divided into two parts. One part occurs in the interval from pouring to the time when the solid fraction in the outer shell of casting reaches 0.7, which is caused by the ferrostatic pressure and can be fed by liquid iron usually. Another part takes place after the solid fraction in the outer shell grows up to 0.7, which is caused by the swelling of the outer shell, and is denoted as V_{mt} here.

3) Calculating the enlargement caused by one of the mesh elements in the outer shell of the casting. The number of mesh elements in the outer shell is denoted as N. If the mesh elements in the shell are in the same size, the contribution of each element to the enlargement caused by the outer shell swelling is V_{mt}/N. If the

swelling of the outer shell is proportional to the solid fraction in the shell, the mold cavity enlargement caused by the element in the shell after the feeding channel is blocked can be expressed as follows,

$$V_{\mathrm{m}}(i)=\begin{cases}\dfrac{V_{\mathrm{mt}}}{N} & f_{\mathrm{S}}(i)\leq 0.7\\[2ex] \dfrac{1.0-f_{\mathrm{S}}(i)}{1.0-0.7}\times\dfrac{V_{\mathrm{mt}}}{N} & f_{\mathrm{S}}(i)>0.7\end{cases} \quad (7)$$

where $f_{\mathrm{S}}(i)$ is the solid fraction of the element when the feeding channel is just blocked.

4) Calculating the mold cavity enlargement occurring after the blocking of feeding channel by accumulating the enlargement caused by each element in the outer shell,

$$V_{\mathrm{m}}=\sum_{i=1}^{N}V_{\mathrm{m}}(i) \quad (8)$$

2 Calculation, Experiment, and Discussion

A real casting was chosen to verify the reliability of the model. The shape and dimensions of the casting and its riser are shown in **figure 3**. In real production, the

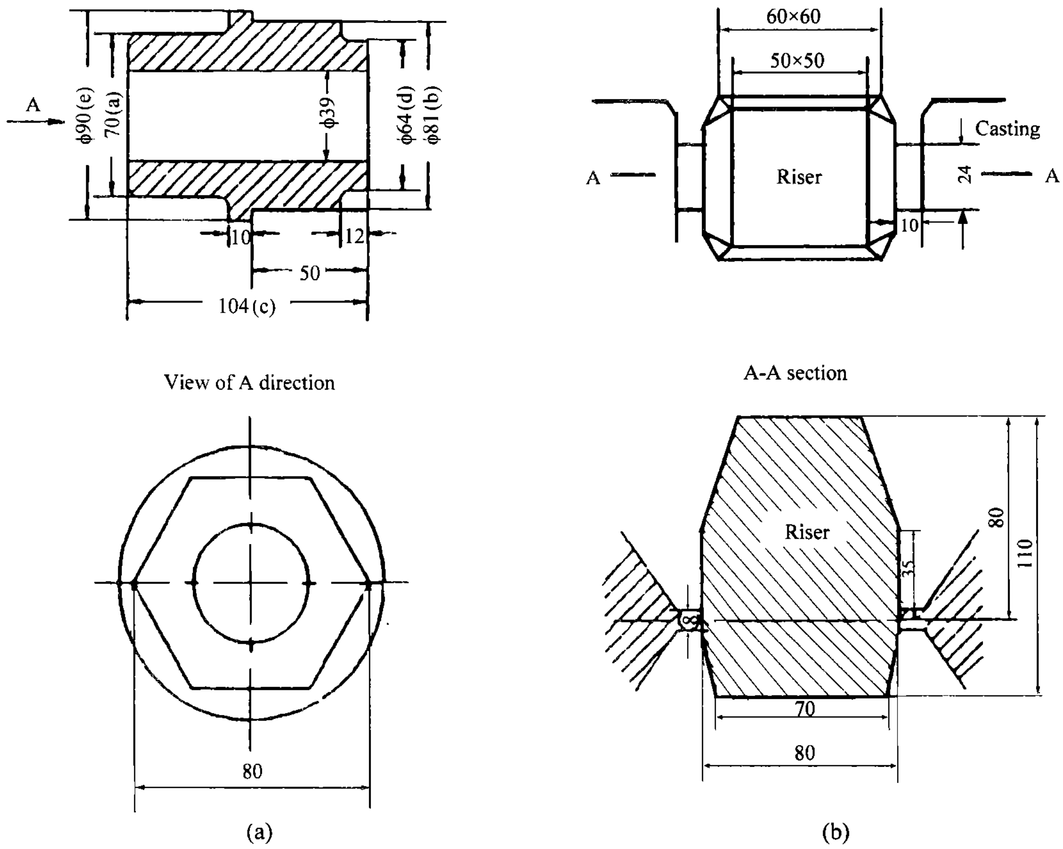

Figure 3 Casting (a) and its riser (b), a–e represent the measured positions. Unit: mm.

green sand mold was used. The hardness value of the mold was in the range of 70–84 (measured by A type sclerometer). The composition of the iron (mass fraction in %) is:C, 3.5–3.9; Si, 2.5–2.8; Mn, 0.3; Mg, 0.055; RE, 0.045. In order to obtain the total amount of mold cavity enlargement in whole solidification process, 10 castings were randomly selected from foundry shop and the dimensions of a, b, c, d, and e (as shown in figure 3 (a)) in the castings were measured at several positions. The dimensions of the castings were compared with those of the pattern, and the solid linear contraction after solidification was considered as 1.0%. Then the total amount of mold cavity enlargement in whole solidification was determined as in the range of 8.78–13.95 cm^3. The mold cavity enlargement caused by the shell swelling, V_{mt}, was taken as 60% of the total enlargement by referring to references [7, 8], and was in the range of 7.28–10.50 cm^3,and the average value was 8.94 cm^3.

At first, the average value of V_{mt}, 8.94 cm^3, is used in the simulation. The values of the main parameters used in the simulation are listed in **table 1**. The obtained plots of volume variations versus time at which the feeding channel is blocked are shown in **figure 4**. In the real casting, the time when the channel is just blocked, t_{CS}, is determined as 196 s by using the model. It is found in

Table 1 The values of the main parameters used in the simulation

Parameter	Value
Temperature at the beginning of eutectic transformation	1 150 ℃
Temperature at the end of eutetic transformation	1 090 ℃
Liquid contraction rate for 100 ℃ temperature drop	−1.8%
Volume variation rate during eutectic transformation	1.7%
Critical solid fraction for juding the blocking of feeding channal	0.7
Thickness of the outer shell of the casting	3.00 mm

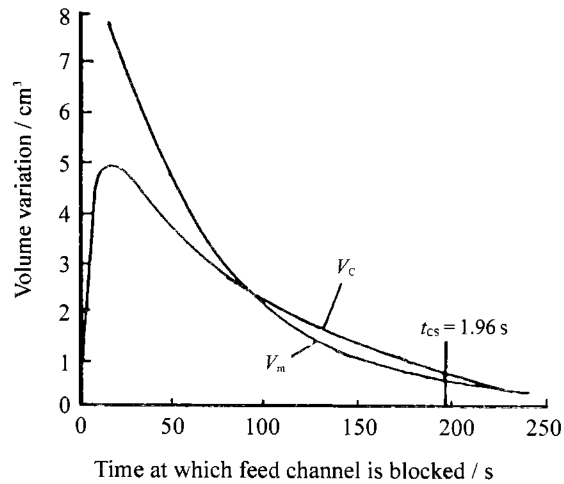

Figure 4 Curves of the volume variations versus the time at which the feeding channel is blocked (V_{mt} = 8.94 cm³)

figure 4 that V_C is larger than V_m and the casting will be sound. However, If the maximum value of V_{mt}, 10.50 cm³, is used in the simulation, V_C is slightly less than V_m as shown in **figure 5**. Under the real production condition, the mold hardness is in a rather wide range, and the V_{mt} values of some castings maybe exceed

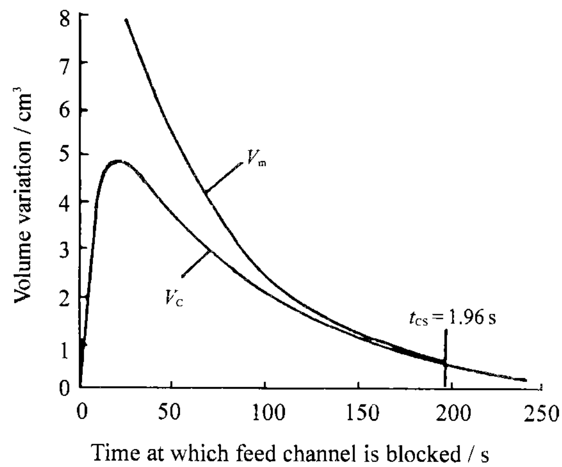

Figure 5 Curves of the volume variations versus the time at which the feeding channel is blocked (V_{mt} = 10.50 cm³)

10.50 cm³. Therefore, a reject rate of 6.2% still exists in real production due to shrinkage defect.

Moreover, in order to verify the effect of mold rigidity on the soundness of casting, 16 castings were made for test by using sodium silicate sand mold. The mold hardness was larger than 90. Because of the increasing of mold hardness, the mold cavity enlargement caused by ferrostatic pressure should be decreased. Thus, the mold cavity enlargement caused by the outer shell swelling was taken as 80% of the total enlargement. By means of the method mentioned above, the values of V_{mt} were determined as 5.41–5.74 cm³. The average value of V_{mt}, 5.54 cm³, is used in the simulation. **Figure 6** shows the obtained results. It is found that V_C significantly exceeds V_m. Since the average value of V_{mt}, 5.54 cm³, is near the maximum value, 5.74 cm³, it can be predicted that the shrinkage defect does not appear. As the prediction, all the 16 castings are free from the defect.

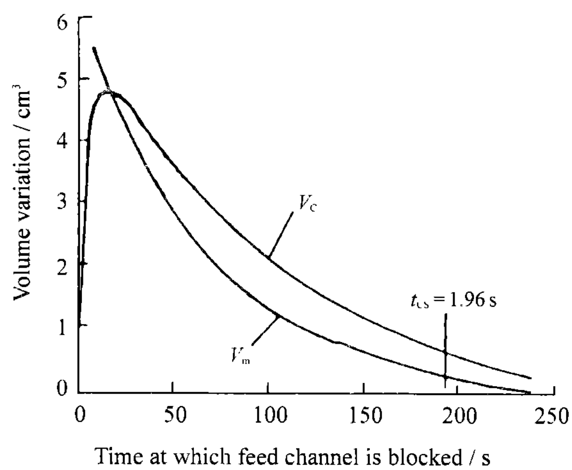

Figure 6 Curves of the volume variations versus the time at which the feeding channel is blocked (V_{mt} = 5.54 cm³)

From the above, it is shown that the results of the model basically coincide with those of production and experiment.

3 Conclusions

(1) The shrinkage defect of a ductile iron casting is attributed to the volume variations occurring in solidification, which consist of liquid contraction, solidification shrinkage, graphitization expansion, and mold cavity enlargement. If the total amount of liquid con-

traction, solidification shrinkage, and graphitization expansion is larger than that of the mold cavity enlargement after the feeding channel is blocked, the shrinkage defect will not be formed.

(2) The shrinkage defect of a ductile iron casting can be effectively predicted by numerically simulating the volume variations occurring in solidification.

(3) A method for quantitatively calculating the mold cavity enlargement occurring after the blocking of feeding channel is presented. In the method, however, a lot of measuring and statistic work must be done. This problem should be improved in the future.

(4) The results of the numerical simulation coincide with those of the production and experiment. The reliability of the present model is verified basically.

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