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Application of Gray Linear Programming in Sintering Mixing Calculation

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Abstract: The mixing calculation cannot be restrained by the quantity of return mines. In order to solve this problem, a method that the sintering mixing proportion is optimized by gray linear programming is presented based on the gray system theory and optimal theory. By using this method, the quality of sintering mines is improved and the energy consumption is reduced.

Key words: gray linear programming; sintering; mixing calculation

In sintering industrial process, there are a series of chemical reactions and physical changes. Under the condition of multitudinous variables, the quality control of sintering mines is very difficult. Aiming at this complex control system, this paper presents a method to improve the quality of sintering mines and to reduce energy consumption based on the gray system theory.

1 Mathematical Model [1–3]

An optimal problem is determined by variables, restraints and object functions. Gray linear programming is to solve the optimal value of the linear function of each variable when a group of linear restraints and limiting condition of nonnegative variables are satisfied.

Definition: Let x_j (j=1,2,$\cdots$,m) be the decision variable, $\boldsymbol{b}_i$ (i=1,2,$\cdots$,N) be the restraint value; $b_i(k)$ be the value of $\boldsymbol{b}_i$ at a given time; and $\boldsymbol{b}_i=(b_i(1),b_i(2),\cdots,b_i(n))$, $i\in I$ be the restraint time series, where I is the object set. Moreover, $\boldsymbol{A}=(a_{ij})$ (i=1,2,$\cdots$,N;j=1,2,$\cdots$,m), $\boldsymbol{C}=(C_1,C_2,\cdots,C_N)$, $\boldsymbol{b}=(b_1,b_2,\cdots,b_N)^{\mathrm{T}}$ at a time of $n+k$. If $b_i(n+k)$ is obtained by GM(1,1) model, the following equations are called gray forecast programming.

$$\begin{cases} \mathrm{GM}:\{\boldsymbol{b}_i\}\to\{\hat{b}_i^{(1)}(n+k)\} \\ \mathrm{IAGO}\{\hat{b}_i^{(1)}(n+k)\}=b_i(n+k)\} \\ \boldsymbol{AX}(n+k)\le\boldsymbol{b}(n+k) \\ \boldsymbol{X}(n+k)\ge 0 \\ f=\boldsymbol{CX}^{\mathrm{T}}(n+k)=\max\ (\text{or}\ \min) \end{cases} \tag{1}$$

$$\text{or}\quad \begin{cases} \sum_j a_{ij}x_j(n+k)\le b_i(n+k) \\ x_j(n+k)\ge 0 \\ f=\sum_j C_j x_j=\max\ (\text{or}\ \min) \end{cases} \tag{2}$$

The gray forecast is that the development and change of characteristic values of the system behavior is forecasted by GM(1,1) model. In this model, the stochastic process is as gray process and the stochastic variables are as gray variables. Modeling steps of the gray forecast is built as follows.

1) Choose a subsequence $\boldsymbol{x}^{(0)}=(x^{(0)}(1),\ x^{(0)}(2),\ \cdots,\ x^{(0)}(N))$ and accumulate this subsequence. $\boldsymbol{x}^{(1)}=(x^{(1)}(1),\ x^{(1)}(2),\cdots,x^{(1)}(N))$ is obtained, where

$$x^{(1)}(t)=\sum_{k=1}^{t}x^{(0)}(k) \tag{3}$$

2) The accumulating matrix $\boldsymbol{B}$ and constant vector $\boldsymbol{Y}_N$ are structured as

$$\boldsymbol{B}=\begin{bmatrix} -0.5(x^{(1)}(1)+x^{(1)}(2)) & 1 \\ -0.5(x^{(1)}(2)+x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -0.5(x^{(1)}(1)+x^{(1)}(1)) & 1 \end{bmatrix} \tag{4}$$

$$\boldsymbol{Y}_N=(x^{(0)}(2),x^{(0)}(3),\cdots,x^{(0)}(N))^{\mathrm{T}} \tag{5}$$

3) The gray parameter $\hat{\boldsymbol{a}}$ is solved by the least-square method. That is

$$\hat{\boldsymbol{a}}=(\boldsymbol{a},u)^{\mathrm{T}}=(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B})^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{Y}_N \tag{6}$$

4) Substitute the time function for gray parameters, we can obtain the following equation:

$$\boldsymbol{X}^{(1)}(t+1)=[\boldsymbol{X}^{(0)}(1)-u/\boldsymbol{a}]\,\mathrm{e}^{-at}+u/\boldsymbol{a} \tag{7}$$

5) The forecast function is obtained by deriving $\hat{\boldsymbol{X}}^{(1)}(k+1)$.

$$\hat{\boldsymbol{X}}^{(0)}(t+1)=-\boldsymbol{a}[\boldsymbol{X}^{(0)}(1)-u/\boldsymbol{a}]\,\mathrm{e}^{-at} \tag{8}$$

$$\text{or}\ \hat{\boldsymbol{X}}^{(0)}(t+1)=\hat{\boldsymbol{X}}^{(1)}(t+1)-\hat{\boldsymbol{X}}^{(0)}(t) \tag{9}$$

6) The difference between $X^{(0)}(t)$ and $\hat{X}^{(0)}(t)$ and the relative error $q(t)$ are taken.

$$e(t)=X^{(0)}(t)-\hat{X}^{(0)}(t) \tag{10}$$

$$q(t)=e^{(0)}(t)/X^{(0)}(t) \tag{11}$$

7) The gray forecast model can be built by the above steps.

Similarly, according to the above steps, the GM(1,1) model can be built for every subsequence, and the gray parameters $\hat{a}$ and u are obtained, where $\hat{a}$ is the development gray number of the system, u is the internal-born control gray number of the system, X is the forecast gray number. In practice application it is not necessary to build GM(1,1) model for all subsequence.

The quantity of sintering return mines in sintering mix is a typical gray quantity. In order to reduce the gray degree of forecast return mines, it is necessary that new information is continually added so that instantaneous information is fully utilized. Consequently, the white degree of return mines rises, and the forecast precision can be improved.

2 Calculation and Analysis [1,2,4]

Table 1 shows the composition of raw materials and their price. The object function is that production cost is the cheapest.

The quantity of sintering return mines in table 1 is determined by the gray forecast. Its data and the forecast value are shown in **table 2** while N=10. According to the given data, the GM(1,1) model can be built as

$$\hat{X}^{(0)}(t+1)=0.005\,3\,(X^{(0)}(1)+37.896\,7/0.005\,3)\,e^{0.005\,3t} \tag{12}$$

Based on the composition of raw materials, their price and the forecast value of sintering return mines, the mix is optimized by the modified simple method. The program flow diagram is shown in **figure 1**.

Table 1 Composition of raw materials and their price

Category of mines		Component mass fraction / %							Price / RMB￥·t^{-1}
		SiO_2	CaO	MgO	Fe	C	FeO	Ig	
Concentrate 1	x_1	9.86	0.30	0.27	0.40	0.00	0.40	0.82	138.60
Concentrate 2	x_2	0.82	0.25	0.21	63.81	0.00	8.53	0.45	104.00
Gas ash	x_3	10.04	8.57	2.80	44.37	13.00	11.30	14.62	0.00
Mizing mines	x_4	9.06	4.50	2.52	52.50	4.07	8.70	5.26	68.50
Concentrate 3	x_5	5.94	5.85	2.91	38.35	0.00	37.90	25.72	52.00
...		...	...	...	...	...	...	...	...
Return mines	x_{12}	9.20	11.96	4.58	52.00	4.43	10.37	0.13	4.43

Note: Ig is the burning.

Table 2 Data of return mines and forecast value

k	1	2	3	4	5	6
Q_t/ kg	40.311	39.788	35.337	37.572	39.383	41.660
k	7	8	9	10	Forecast value	
Q_t/ kg	39.766	40.130	38.456	39.211	40.078	

Note: Q_t is the quantity of return mines.

The restraint of every given sintering quantity is shown in **table 3** as well as the optimal solution listed in **table 4** can be calculated according to the above steps and program flow diagram. In addition, **table 5** shows the comparison between the real consumption of sintering mines and the consumption of the optimal mix per ton.

From **table 6**, it is can be seen that RMB￥2.87/t is reduced when the cost of the raw materials of sintering mines is compared with the real one per ton. If the yield is 500 000 t/year, the economical result can increase RMB￥1 435 000 /year.

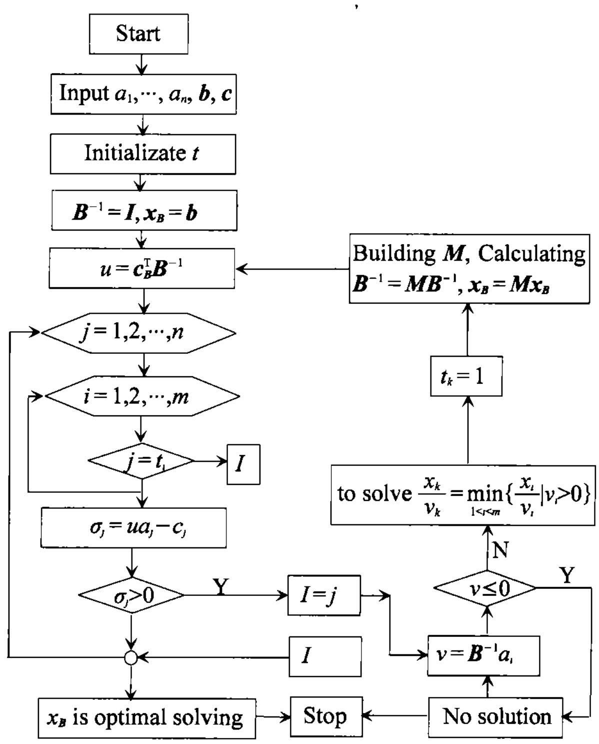

Figure 1 Modified simple method and its program flow diagram.

Changing the producing condition and external environment, the decision variable or restraint condition can be increased or decreased by the model. When the value of the setting parameter is different, many schemes can be compared. An optimal scheme is chosen from them.

Table 3 Restraint of given sintering quantity %

x	Maximum	Minimum
1	35.00	20.00
2	1.50	8.00
3	3.30	3.00
4	1.10	1.00
5	1.20	1.00
6	2.20	2.00
7	10.00	0.00
8	6.00	5.00
9	20.00	10.00
10	1.00	0.50
11	1.20	1.00
12	40.59	36.72

Table 4 Optimal solution of sintering mix

x	Proportion of burden / %	Quantity of burden / kg	cost / RMB¥ $\cdot t^{-1}$
1	0.20	20.00	27.70
2	0.08	8.00	8.32
3	0.03	3.30	0.00
4	0.01	1.00	0.69
5	0.01	1.00	0.52
6	0.02	2.00	0.24
7	0.04	4.35	0.48
8	0.05	5.00	3.55
9	0.18	17.9	1.79
10	0.01	0.62	0.06
11	0.01	1.00	0.11
12	0.37	36.70	22.00
Total	1.00	100.00	65.30

Table 5 Comparison of the consumption of raw materials of sintering mines kg/t

Category of mines	Consumption		
	Real	Optimal	Error
Concentrate 1	20.90	20.00	−0.89
Concentrate 2	8.04	8.00	−0.04
Gas ash	3.29	3.30	0.01
Mixing mines	1.02	1.00	−0.02
Concentrate 3	0.99	1.00	0.00
Quicklime	1.90	2.00	0.11
Magnesia	5.00	4.36	−0.65
Fuel	6.00	5.00	−1.00
Limestone	18.40	17.90	−0.52
Dust ash	0.00	0.62	0.62
Zip fasteners ash	0.00	1.00	1.00
Return mines	38.00	36.70	1.28
Total	104.00	100.00	−3.51

Table 6 Comparison of the real cost with the cost of calculated optimal mix RMB¥/t

Category of mines	Real cost	Optimal cost	Change value
Concentrate 1	28.95	27.72	1.23
Concentrate 2	8.36	8.32	0.04
Gas ash	0.00	0.00	0.00
Mixing mines	0.69	0.69	0.00
Concentrate 3	0.51	0.52	−0.01
Quicklime	0.23	0.24	−0.01
Magnesia	0.55	0.48	0.07
Fuel	4.26	3.55	0.71
Limestone	1.84	1.79	0.05
Dust ash	0.00	0.06	−0.06
Zip fasteners ash	0.00	0.11	−0.11
Return mines	22.80	22.03	0.77
Total	68.20	65.33	2.67

In order to increase the iron content of sintering mines, the mix quantity of concentrate 1 rises and the restricted condition is changed to $35\% \le x_1 \le 25\%$. Then the optimal result of the mix is shown as in **table 7**. In this optimization, the iron content of sintering mines is improved, the price rises too (see **table 8**).

In order to increase the quantity of return mines, we add the mix quantity of return mines. Then the restricted condition is changed to $38.1\% \le x_{12} \le 42.11\%$. The optimal result of the mix is shown as in **table 9**. In this optimization, the iron content of sintering mines is increased and the price rises too. Meanwhile, the quantity of the return mines can be improved (see **table 10**).

In order to ensure that the sintering industrial process goes on smoothly, the carbon content is increased and the restricted condition is changed to $9\% \le x_8 \le 6\%$. In addition, the content of concentrate mine 1 is reduced as well as the restricted condition is changed to $30\% \le x_1 \le 20\%$. Then the optimal result of the mix is shown as in **table 11**, and the attained target is shown as in **table 12**.

Table 7 Optimal result of sintering mix for increasing the iron content of sintering mines

x	1	2	3	4
Proportion of burden / %	25.00	8.00	3.00	1.00
x	5	6	7	8
Proportion of burden / %	1.00	2.00	4.22	5.00
x	9	10	11	12
Proportion of burden / %	19.77	0.50	1.00	36.72

Table 8 Attained target of sintering mix for increasing the iron content of sintering mines

Component mass fraction / %				Price / RMB￥·t^{-1}
MgO	Fe	C	R	
4.50	42.91	5.00	2.00	72.44

Note: *R* denotes the basicity concentration.

Table 9 Optimal result of sintering mix for increasing the quantity of return mines

x	1	2	3	4
Proportion of burden / %	25.00	8.00	3.00	1.00
x	5	6	7	8
Proportion of burden / %	1.00	2.00	4.07	5.00
x	9	10	11	12
Proportion of burden / %	19.95	0.05	1.00	38.10

Table 10 Attained target of sintering mix for increasing the quantity of return mines

Component mass fraction / %				Price / RMB￥·t^{-1}
MgO	Fe	C	R	
4.50	43.62	5.00	5.00	73.27

Table 11 Optimal result of sintering mix for ensuring that the sintering industrial process goes on smoothly

x	1	2	3	4
Proportion of burden / %	20.00	12.53	3.30	1.00
x	5	6	7	8
Proportion of burden / %	1.00	2.20	1.00	6.00
x	9	10	11	12
Proportion of burden / %	2.00	0.50	1.00	38.10

Table 12 Attained target of sintering mix for increasing the quantity of return mines

Component mass fraction / %				Price / RMB￥·t^{-1}
MgO	Fe	C	R	
7.25	43.62	2.00	6.00	72.44

As seen as table 12, the Fe content of sintering mines is increased and the price rises too. However, the content of concentrate mines is reduced. So the sintering process is tending towards stability.

3 Conclusions

A new method of sintering mixing calculation is proposed based on gray linear programming. The variance tendency of the quantity of return mines is analyzed by the method in sintering process, and the foundation is provided by the model for adding the restraint of return mines in sintering mixing calculation. The balance of raw materials is ensured while the restraint of return mines is added in sintering mixing calculation, and the sintering mixing proportion is optimized. The flexibility of mixing calculation rises greatly as well as the quality of sintering mines is improved. The problem, which the quantity of return mines cannot be controlled, was solved. In a word, the quality of sintering mines is improved and the energy consumption is reduced.

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