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Information

Analytical Criteria for Local Activity of CNN with Two Ports and Application to Smoothed Chua's Circuit

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Abstract: Presents analytic criteria for the local activity theory in two-port cellular neural network (CNN) cells with four local state variables, and gives the application to a smoothed Chua's circuit (SCC) CNN with two-port and 15×15 arrays. The bifurcation diagrams of the SCC CNN show that they are completely the same as those of an SCC CNN with one-port calculated earlier, which do not exist locally passive domain. The evolution of the patterns of the state variables of the SCC CNN is stimulated. Oscillatory patterns, chaotic patterns, or divergent patterns may emerge if the selected cell parameters are located in the locally active unstable domains but nearby the edge of chaos domain.

Key words: cellular neural network; local activity; smoothed Chua's circuit; two-port; chaos

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1 Introduction

The CNN is a multi-disciplinary area first introduced by Chua and Yang in 1988 [1, 2] after Hopfield proposed his fully-connected neural networks [3]. Hopfield network grows exponetially with the size of the array, which cannot be built even in modest array size. However, CNN can be built to be no electrical interconnections beyond a prescribed sphere of influnce. Consequently, CNNs have been widely studied for both theoretic foundations [1, 4–11] and image proccessing [2, 12–18].

CNN can also be used for the study of complex patterns and structures,emerging from homogeneouse media operating far from thermodynamic equilibrium [17,19]. In particular, the analytical criteria for local activity of CNNs have been applied successfully to the study of the dynamics of the CNNs [20–27] related to the Fitzhugh-Nagumo equation, Brussselator equation, Gierer-Meinhart equation, Oregonator equation, Hodgkin-Huxley equation, enzyme reaction equation, and the smoothed Chua's circuit.

In this paper, the analytic criteria for testing the local activity of the CNN's with four state variables and two ports are set up. As an application of the analytic criteria, a modified dual-layer two-dimensional reaction-diffusion CNN with two ports and 15×15 arrays of SCC CNN has been introduced. The bifurcation diagrams of the SCC CNN have been calculated, which are completely the same as those of the modified dual-layer two-dimensional reaction-diffusion CNN with one port. The numerical calculations show that the trajectories of the 900 local state variables of the SCC CNN can exihibit oscillatory patterns, chaotic patterns, or divergent patterns if the selected cell parameters are located in the locally active unstable domains but nearby the edge of chaos domain.

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2 Analytical Critria of Local Activity

For a two-port CNN cell, the corresponding local state equations with "4" state variables assume the form

$$\dot{\boldsymbol{V}}_a=\boldsymbol{A}_{aa}\boldsymbol{V}_a+\boldsymbol{A}_{ab}\boldsymbol{V}_b+\boldsymbol{I}_a, \tag{1}$$

$$\dot{\boldsymbol{V}}_b=\boldsymbol{A}_{ba}\boldsymbol{V}_a+\boldsymbol{A}_{bb}\boldsymbol{V}_b \tag{2}$$

where

$$\boldsymbol{V}_a=[V_1,V_2]^T,\ \boldsymbol{V}_b=[V_3,V_4]^T,\ \boldsymbol{I}_a=[I_1,I_2]^T;$$

$$\boldsymbol{A}_{aa}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix},\ \boldsymbol{A}_{ab}=\begin{bmatrix}a_{13} & a_{14}\\ a_{23} & a_{24}\end{bmatrix},$$
$$\boldsymbol{A}_{ba}=\begin{bmatrix}a_{31} & a_{32}\\ a_{41} & a_{42}\end{bmatrix},\ \boldsymbol{A}_{bb}=\begin{bmatrix}a_{33} & a_{34}\\ a_{43} & a_{44}\end{bmatrix}. \tag{3}$$

The corresponding CNN cell impedance $Z_Q(s)$ is given by reference [17]:

$$Z_Q^{-1}(s)=Y_Q(s)$$
$$=(s\boldsymbol{I}-\boldsymbol{A}_{aa})-\boldsymbol{A}_{ab}(s\boldsymbol{I}-\boldsymbol{A}_{bb})^{-1}\boldsymbol{A}_{ba}$$

$$=\begin{bmatrix} s-a_{11} & -a_{12} \\ -a_{21} & s-a_{22}\end{bmatrix}-$$

$$\frac{1}{s^2+a_{00}s+b_{00}}\begin{bmatrix} q_{11}\ s+q_{110} & q_{12}\ s+q_{120} \\ q_{13}\ s+q_{130} & q_{14}\ s+q_{140}\end{bmatrix} \tag{4}$$

where

$$q_{11}=a_{13}a_{31}+a_{14}a_{41}, \tag{5}$$

$$q_{110}=a_{13}a_{34}a_{41}+a_{14}a_{43}a_{31}-a_{13}a_{31}a_{44}-a_{41}a_{14}a_{33}, \tag{6}$$

$$q_{12}=a_{13}a_{32}+a_{14}a_{42}, \tag{7}$$

$$q_{120}=a_{13}a_{34}a_{42}+a_{14}a_{32}a_{43}-a_{13}a_{32}a_{44}-a_{14}a_{33}a_{42}, \tag{8}$$

$$q_{13}=a_{23}a_{31}+a_{24}a_{41}, \tag{9}$$

$$q_{130}=a_{24}a_{31}a_{43}+a_{23}a_{34}a_{41}-a_{23}a_{31}a_{44}-a_{24}a_{33}a_{41}, \tag{10}$$

$$q_{14}=a_{23}a_{32}+a_{24}a_{42}, \tag{11}$$

$$q_{140}=a_{23}a_{34}a_{42}+a_{24}a_{32}a_{43}-a_{23}a_{32}a_{44}-a_{24}a_{33}a_{42}, \tag{12}$$

$$a_{00}=-a_{33}-a_{44},\ b_{00}=a_{33}a_{44}-a_{34}a_{43}. \tag{13}$$

Lemma 1 [17] A two-port Reaction Diffusion CNN cell is locally active at a cell equilibrium point $Q\triangleq(\overline{V}_1, \overline{V}_2, \overline{I}_1, \overline{I}_2)$ if, and only if, its cell impedance at Q satisfies at least one of the following 4 conditions:

(1) $Y_Q(s)$ has a pole in $\mathrm{Re}[s]>0$.

(2) $Y^H{}_Q(\mathrm{i}\omega)=Y^{\dagger}{}_Q(\mathrm{i}\omega)+Y_Q(\mathrm{i}\omega)$ is not a positive semi-definite matrix at some $\omega=\omega_0$ where ω_0 is any real number, and $Y^{\dagger}{}_Q(s)$ is constructed by taking first the transpose of $Y_Q(s)$, and then followed by the complex conjugate operation.

(3) $Y_Q(s)$ has a simple pole $s=\mathrm{i}\omega_p$ on the imaginary axis,where its associated residue matirx

$$k_1\triangleq\begin{cases}\lim\limits_{s\to \mathrm{i}\omega_p}(s-\mathrm{i}\omega_p)Y_Q(s), & \text{if } \omega_p<\infty \\ \lim\limits_{\omega_p\to\infty}Y_Q(\mathrm{i}\omega_p)/\mathrm{i}\omega_p, & \text{if } \omega_p=\infty\end{cases}$$

is either not a Hermitian matrix, or else not a positive semi-definite Hermitian matrix.

(4) $Y_Q(s)$ has a multiple pole on the imaginary axis.

Theorem 1 For $i=1,2,3,4$, let q_{1i}, q_{1i0}, a_{00} and b_{00} be defined by formulas (5)–(13). Then $Y_Q(s)$ satisfies condition 1 in Lemma 1 if, and only if at least one of the following conditions holds:

(1) $a_{00}>0,\ a_{00}^2-4b_{00}\le 0$.

(2) $s=\dfrac{a_{00}+\sqrt{a_{00}^2-4b_{00}}}{2}>0$, and there exists an $i\in\{1,2,3,4\}$ s.t. $q_{1i0}-q_{1i}s\neq 0$.

(3) $s=\dfrac{a_{00}-\sqrt{a_{00}^2-4b_{00}}}{2}>0$, and there exists an $i\in\{1,2,3,4\}$ s.t. $q_{1i0}-q_{1i}s\neq 0$.

Theorem 2 For $i=1,2,3,4$, let q_{1i}, q_{1i0}, a_{00} and b_{00} be defined by formulas (5)–(13). Then $Y_Q(s)$ satisfies condition 3 in Lemma 1 if, and only if, at least one of the following condition holds.

(1) If $a_{00}\neq 0$, $b_{00}=0$, $\max\{|q_{110}|, |q_{120}|, |q_{130}|, |q_{140}|\}\neq 0$, and

(a) $q_{120}\neq q_{130}$ or

(b) $q_{120}=q_{130}$ but $q_{110}/a_{00}>0$, or $(q_{110}q_{140}-q_{120}q_{130})/a_{00}>0$.

(2) If $a_{00}=0$, $b_{00}>0$,
$\max\{|q_{11}|+|q_{110}|, |q_{12}|+|q_{120}|, |q_{13}|+|q_{130}|, |q_{14}|+|q_{140}|\}\neq 0$, and

(a) $Y_Q(s)$ does not satisfy the following conditions

$$q_{110}=q_{140}=0,\ q_{12}=q_{13},\ q_{120}=q_{130} \tag{$*$}$$

(b) $Y_Q(s)$ satisfies condition ($*$) and

$q_{11}+q_{14}>0$ or $q_{11}q_{14}-q_{12}^2-q_{120}^2/b_{00}<0$.

Theorem 3 For $i=1,2,3,4$, let q_{1i}, q_{1i0}, a_{00} and b_{00} be defined by formulas (5)–(13). Then $Y_Q(s)$ has a multiple pole on the imaginary axis if, and only

$a_{00}=b_{00}=0$, and $\max\{|q_{110}|,|q_{120}|,|q_{130}|,|q_{140}|\}\neq 0$.

In order to set up the analytic criteria for condition 2 in Lemma 1, the following equailities need to be given in advance:

$$A_{11}=2(a_{00}q_{11}-q_{110});\ B_{11}=2b_{00}q_{110}, \tag{14}$$

$$A_{12}=a_{00}(q_{13}+q_{12})-q_{130}-q_{120}, \tag{15}$$

$$B_{12}=b_{00}(q_{120}+q_{130}), \tag{16}$$

$$B_{120}=b_{00}(q_{12}-q_{13})+a_{00}(q_{130}-q_{120}), \tag{17}$$

$$A_{14}=2(a_{00}q_{14}-q_{140}), \tag{18}$$

$$B_{14}=2b_{00}q_{140}. \tag{19}$$

$$\Omega_1=-\frac{2a_{00}^2(a_{11}+a_{22})+A_{11}+A_{14}}{4(a_{11}+a_{22})} \tag{20}$$

$$g_1(\Omega_1)=2(a_{11}+a_{22})\Omega_1^2+[2a_{00}^2(a_{11}+a_{22})+A_{11}A_{14}]\Omega_1+B_{11}+B_{14}. \tag{21}$$

$$\Omega_2=-\frac{A_{11}+A_{14}}{4(a_{11}+a_{22})}, \tag{22}$$

$$g_2(\Omega_2)=2(a_{11}+a_{22})\Omega_2^2+(A_{11}+A_{14})\Omega_2+B_{11}+B_{14}. \tag{23}$$

$$a_1^*=\frac{2(B_{11}+B_{14})}{A_{11}+A_{14}}, \tag{25}$$

$$a_0^*=-b_{00}^2+\frac{(a_{00}^2-2b_{00})(B_{11}+B_{14})}{A_{11}+A_{14}}. \tag{26}$$

$$\Omega_3=\frac{-a_1^*+\sqrt{a_1^{*2}-4a_0^*}}{2}; \tag{27}$$

$$\Omega_4=\frac{-a_1^*-\sqrt{a_1^{*2}-4a_0^*}}{2}; \tag{28}$$

$$g_3(\Omega_i)=2(a_{11}+a_{22})+\frac{\Omega_i(A_{11}+A_{14})+B_{11}+B_{14}}{(b_{00}-\Omega_i)^2+a_{00}^2\Omega_i},\ i=3,4. \tag{29}$$

$$\Omega_5=(2b_{00}-a_{00})/2, \tag{30}$$

$$g_4(\Omega_5)=2(a_{11}+a_{22})[(b_{00}-\Omega_5)^2+a_{00}^2\Omega_5]+B_{11}+B_{14}. \tag{31}$$

$$MM=2b_{00}^2[a_{22}B_{11}+a_{11}B_{14}-(a_{12}+a_{21})B_{12}]+ B_{11}B_{14}-B_{12}^2. \tag{32}$$

$$J=2a_{22}A_{11}+2a_{11}A_{14}-2(a_{12}+a_{21})A_{12}-(q_{12}-q_{13})^2, \tag{33}$$

$$K=2a_{22}B_{11}+2a_{22}(a_{00}^2-2b_{00})A_{11}+2a_{11}B_{14}+2a_{11}(a_{00}^2-2b_{00})A_{14}-2(a_{12}+a_{21})B_{12}+A_{11}A_{14}-2(a_{12}+a_{21})A_{12}(a_{00}^2-2b_{00})-A_{12}^2-2(q_{12}-q_{13})B_{120}; \tag{34}$$

$$L=2a_{22}B_{11}(a_{00}^2-2b_{00})+2a_{22}A_{11}b_{00}^2+2a_{11}B_{14}(a_{00}^2-2b_{00})+2a_{11}A_{14}b_{00}^2-2(a_{12}+a_{21})B_{12}(a_{00}^2-2b_{00})-2(a_{12}+a_{21})A_{12}b_{00}^2-2A_{12}B_{12}-B_{120}^2+B_{11}A_{14}+A_{11}B_{14}, \tag{35}$$

$$M=2a_{22}B_{11}b_{00}^2+2a_{11}B_{14}b_{00}^2-2(a_{12}+a_{21})B_{12}b_{00}^2+B_{11}B_{14}-B_{12}^2. \tag{36}$$

$$\Omega_6=\frac{-K+\sqrt{K^2-3JL}}{3J}; \tag{37}$$

$$\Omega_7=\frac{-K-\sqrt{K^2-3JL}}{3J}; \tag{38}$$

$$g_5(\Omega_i)=J\Omega_i^3+K\Omega_i^2+L\Omega_i+M,\quad i=6,7. \tag{39}$$

$$\Omega_8=-\frac{L}{2K}, \tag{40}$$

$$g_6(\Omega_8)=K\Omega_8^2+L\Omega_8+M. \tag{41}$$

$$E=4a_{11}a_{22}-(a_{12}+a_{21})^2. \tag{42}$$

$$F=2(a_{00}^2-2b_{00})[4a_{11}a_{22}-(a_{12}+a_{21})^2]+J. \tag{43}$$

$$G=(a_{00}^4-4a_{00}^2b_{00}+6b_{00}^2)[4a_{11}a_{22}-(a_{12}+a_{21})^2]+K. \tag{44}$$

$$H=2b_{00}^2(a_{00}^2-2b_{00})[4a_{11}a_{22}-(a_{12}+a_{21})^2]+L. \tag{45}$$

$$I=b_{00}^4[4a_{11}a_{22}-(a_{12}+a_{21})^2]+M. \tag{46}$$

$$w_1=\frac{-1+i\sqrt{3}}{2},\quad w_2=\frac{-1-i\sqrt{3}}{2}. \tag{47}$$

$$\alpha_0=\frac{3F}{4E},\ \beta_0=\frac{G}{2E}. \tag{48}$$

$$p=-\frac{1}{3}\alpha_0^2+\beta_0, \tag{49}$$

$$q=\frac{2}{27}\alpha_0^3-\frac{\alpha_0\beta_0}{3}+\frac{H}{4E}, \tag{50}$$

$$D=\frac{q^2}{4}+\frac{p^3}{27}. \tag{51}$$

$$\Delta_1=\{-q/2+[(q/2)^2+p^3/27]^{1/2}\}^{1/3}+\{-q/2-[(q/2)^2+p^3/27]^{1/2}\}^{1/3}, \tag{52}$$

$$\Delta_2=w_1\{-q/2+[(q/2)^2+p^3/27]\}^{1/2})^{1/3}+w_2\{-q/2-[(q/2)^2+p^3/27]^{1/2}\}^{1/3}, \tag{53}$$

$$\Delta_3=w_2\{-q/2+[(q/2)^2+p^3/27]^{1/2}\}^{1/3}+w_1\{-q/2-[(q/2)^2+p^3/27]^{1/2}\}^{1/3} \tag{54}$$

$$\Omega_9=\Delta_1-\alpha_0/3, \tag{55}$$

$$\Omega_{10}=\Delta_2-\alpha_0/3, \tag{56}$$

$$\Omega_{11}=\Delta_3-\alpha_0/3, \tag{57}$$

$$\Omega_{12}=\frac{-G+\sqrt{G^2-3FH}}{3F}, \tag{58}$$

$$\Omega_{13}=\frac{-G-\sqrt{G^2-3FH}}{3F}, \tag{59}$$

$$\Omega_{14}=-\frac{H}{G}, \tag{60}$$

$$g_7(\Omega_i)=E\Omega_i^4+F\Omega_i^3+G\Omega_i^2+H\Omega_i+I,\quad i=9,\ 10,\cdots,14. \tag{61}$$

Theorem 4 $Y_Q(s)$ satisfies condition 2 in Lemma 1, if and only if one of the following conditions holds (The meanings of the parameters are the same as those given in formulas (14)–(61)).

(i) $a_{11}+a_{22}>0$.

(ii) $a_{11}+a_{22}=0$ and $A_{11}+A_{14}>0$.

(iii) $a_{11}+a_{22}=0$, $A_{11}+A_{14}\le 0$ and $B_{11}+B_{14}>0$.

(iv) $a_{11}+a_{22}<0$, $b_{00}\ne 0$ and $2(a_{11}+a_{22})+\frac{B_{11}+B_{14}}{b_{00}^2}>0$.

(v) $a_{11}+a_{22}<0$, $b_{00}=0$, $a_{00}\ne 0$ and $B_{11}+B_{14}>0$.

(vi) $a_{11}+a_{22}<0$, $b_{00}=0$, $a_{00}\ne 0$, $B_{11}+B_{14}\le 0$, $\Omega_1\ge 0$, $g_1(\Omega_1)>0$.

(vii) $a_{11}+a_{22}<0$, $a_{00}=0$, $b_{00}=0$, $\Omega_2\ge 0$, $g_2(\Omega_2)>0$.

(viii) $a_{11}+a_{22}<0$, $a_{00}\ne 0$, $b_{00}\ne 0$, $A_{11}+A_{14}\ne 0$ and

(a) $\Omega_3\ge 0$, $g_3(\Omega_3)>0$ or

(b) $\Omega_4\ge 0$, $g_3(\Omega_4)>0$.

(ix) $a_{11}+a_{22}<0$, $a_{00}\ne 0$, $b_{00}\ne 0$, $A_{11}+A_{14}=0$, $\Omega_5\ge 0$, $g_4(\Omega_5)>0$.

(x) $4a_{11}a_{22}-(a_{12}+a_{21})^2<0$.

(xi) $4a_{11}a_{22}-(a_{12}+a_{21})^2=0$ and $MM<0$.

(xii) $4a_{11}a_{22}-(a_{12}+a_{21})^2=0$, $MM\ge 0$, $J\ne 0$ and

(a) $\Omega_6\ge 0$, $g_5(\Omega_6)<0$ or

(b) $\Omega_7\ge 0$, $g_5(\Omega_7)<0$.

(xiii) $4a_{11}a_{22}-(a_{12}+a_{21})^2=0$, $MM\ge 0$, $J=0$, $K\ne 0$, $\Omega_8\ge 0$, and $g_6(\Omega_8)<0$.

(xiv) $4a_{11}a_{22}-(a_{12}+a_{21})^2=0$, $MM\ge 0$, $J=K=0$, and (a) $M<0$ or (b) $L<0$.

(xv) $4a_{11}a_{22}-(a_{12}+a_{21})^2>0$, and

(a) $D>0$, $\Omega_9\ge 0$, $g_7(\Omega_9)<0$, or

(b) $D<0$, and

(1) $\Omega_9\ge 0$, $g_7(\Omega_9)<0$ or

(2) $\Omega_{10} \geq 0$, $g_7(\Omega_{10}) < 0$ or

(3) $\Omega_{11} \geq 0$, $g_7(\Omega_{11}) < 0$.

(c) $D = 0$, $p = q = 0$ and $\alpha<0$, $g_7(-\alpha_0/3) < 0$.

(d) $D = 0$, $q^2/4 = -p^3/27 \neq 0$ and

(1) $\Omega_9 \geq 0$, $g_7(\Omega_9) < 0$ or

(2) $\Omega_{10} \geq 0$, $g_7(\Omega_{10}) < 0$.

2 SCC CNN and Its Bifurcation Diagrams

Firstly, an SCC CNN with two ports and 15×15 arrays is introduced which is similar to the one given in [28]. It has been found that these SCC CNN's might be the most friendly models, which either have very complex dynamical behaviors or are studied easily via the local activity principle.

The SCC Equations is represented as follows.

$$\frac{dx_1}{dt} = \alpha[x_2 - x_1 - bx_1 - \frac{(a-b)}{\pi}\arctan(5x_1)] \tag{62}$$

$$\frac{dx_2}{dt} = x_1 - x_2 + x_3 \tag{63}$$

$$\frac{dx_3}{dt} = -\beta x_2 \tag{64}$$

$$\frac{dx_4}{dt} = \alpha[x_1 - x_4 - 2bx_1 - \frac{2(a-b)}{\pi}\arctan(5x_1)] \tag{65}$$

where α, β, a and b are parameters.

Now let us map the SCC Equations into a 2-dimensional 15×15 SCC CNN with two ports (two diffusion constants D_1 and D_2):

$$\dot{x}_{1i,j} = \alpha[x_{2i,j} - x_{1i,j} - bx_{1i,j} - \frac{(a-b)}{\pi}\arctan(5x_{1i,j})] + D_1[x_{1i+1,j} + x_{1i-1,j} + x_{1i,j+1} + x_{1i,j-1} - 4x_{1i,j}] \tag{66}$$

$$\dot{x}_{2i,j} = x_{1i,j} - x_{2i,j} + x_{3i,j} + D_2[x_{2i+1,j} + x_{2i-1,j} + x_{2i,j+1} + x_{2i,j-1} - 4x_{2i,j}] \tag{67}$$

$$\dot{x}_{3i,j} = -\beta x_{2i,j} \tag{68}$$

$$\dot{x}_{4i,j} = \alpha[x_{1i,j} - x_{4i,j} - 2bx_{1i,j} - \frac{2(a-b)}{\pi}\arctan(5x_{1i,j})]$$

$$i,j = 1, 2, \cdots, 15. \tag{69}$$

In component form, Equations (66)–(69) become

$$\dot{X}_1 = f_1(X_1, X_2, X_3, X_4) + D_1\nabla^2 X_1, \tag{70}$$

$$\dot{X}_2 = f_2(X_1, X_2, X_3, X_4) + D_2\nabla^2 X_2, \tag{71}$$

$$\dot{X}_3 = f_3(X_1, X_2, X_3, X_4), \tag{72}$$

$$\dot{X}_4 = f_4(X_1, X_2, X_3, X_4) \tag{73}$$

where

$$f_1(X_1, X_2, X_3, X_4) = \alpha[X_2 - X_1 - bX_1 - \frac{(a-b)}{\pi}\arctan(5X_1)].$$

$$f_2(X_1, X_2, X_3, X_4) = X_1 - X_2 + X_3.$$

$$f_3(X_1, X_2, X_3, X_4) = -\beta X_2.$$

$$f_4(X_1, X_2, X_3, X_4) = \alpha[X_1 - X_4 - 2bX_1 - 2\frac{(a-b)}{\pi}\arctan(5X_1)]$$

and ∇^2 corresponds to a 225×225 matrix.

The cell equilibrium points Q_i's of equations (70)–(73) for the restricted local activity domain [20] can be determined via equations:

$$f_1(X_1, X_2, X_3, X_4) = 0 \tag{74}$$

$$f_2(X_1, X_2, X_3, X_4) = 0 \tag{75}$$

$$f_3(X_1, X_2, X_3, X_4) = 0 \tag{76}$$

$$f_4(X_1, X_2, X_3, X_4) = 0 \tag{77}$$

From Equations (74)–(77), it can be concluded that for any parameter group $\{\alpha, \beta, a, b\}$, there exists at least one cell equilibrium point

$Q_1 = (0, 0, 0, 0)$.

On the other hand, any other cell equilibrium point $Q_i = (x_1, x_2, x_3, x_4)$ must satisfy the following equations.

$$x_2 = 0,\ x_3 = -x_1,$$

$$(1+b)x_1 + \frac{(a-b)}{\pi}\arctan(5x_1) = 0$$

$$x_1 - x_4 - 2bx_1 - 2\frac{(a-b)}{\pi}\arctan(5x_1) = 0.$$

Consequently, it can be obtained by the following constraint condition.

Constraint condition: If Q_2 is a nonzero equilibrium point, then $Q_3 = -Q_2$ is also an equilibrium point, and the parameters a and b have to satisfy the inequality

$$(b+1) < 5(b-a)/\pi \tag{78}$$

Using constraint condition (78), the admissible parameter domain for the real equilibrium points Q_2 and Q_3 are the same as those shown in figure 3 in reference [24]. The cell coefficients $a_{m,n}(Q_i)$'s are defined via the corresponding Jacobian matrix and are also the same as those given in reference [24]:

$$A_i \triangleq \begin{bmatrix} a_{11}(Q_i) & a_{12}(Q_i) & a_{13}(Q_i) & a_{14}(Q_i) \\ a_{21}(Q_i) & a_{22}(Q_i) & a_{23}(Q_i) & a_{24}(Q_i) \\ a_{31}(Q_i) & a_{32}(Q_i), & a_{33}(Q_i) & a_{34}(Q_i) \\ a_{41}(Q_i) & a_{42}(Q_i) & a_{43}(Q_i) & a_{44}(Q_i) \end{bmatrix} = \begin{bmatrix} a_{11}(Q_i) & a_{12}(Q_i) & 0 & 0 \\ a_{21}(Q_i) & a_{22}(Q_i) & a_{23}(Q_i) & 0 \\ 0 & a_{32}(Q_i) & 0 & 0 \\ a_{41}(Q_i) & 0 & 0 & a_{44}(Q_i) \end{bmatrix} \tag{79}$$

where

$$a_{11} = \alpha\{-1 - [b + \frac{5(a-b)}{\pi(1+(5x_1)^2)}]\}, \tag{80}$$

$$a_{12}=\alpha,\ a_{21}=1,\ a_{22}=-1,\ a_{23}=1,\ a_{32}=-\beta, \tag{81}$$

$$a_{44}=-\alpha,\ a_{41}=\alpha\{-1-2[b+\frac{5(a-b)}{\pi(1+(5x_1)^2)}]\} \tag{82}$$

Using Theorems 1–4, contrain condition (78), and formulas (3)–(13), (79)–(82) the locally active domains, locally passive domains and edges of chaos with respect to the equilibrium points Q_1 and Q_2 (resp. Q_3) with different cell parameters are shown in **figures 1**(a) and 1(b), respectively. From these graphs, it can be concluded that:

- For all the selected parameter groups, the corresponding bifurcation diagrams do not have locally passive domains.
- The two bifurcation graphs are the same as those shown in figures 5(a) and (b) in reference [24], respectively.

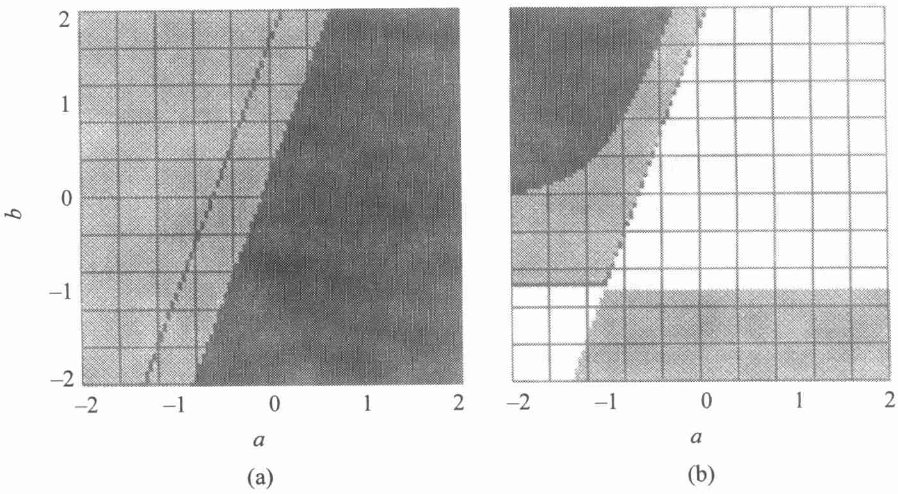


Figure 1 Bifurcation diagrams of the SCC CNN with parameters $\alpha=10$ and $\beta=15$. Edge of chaos domain (black), locally active and unstable domain (gray). In the domain coded with white, no nonzero equilibrium point exists. Equilibrium points (a) Q_1 and (b) Q_2 (*resp.* Q_3).

3 Dynamic simulations of the SCC CNN's

Now let us deal with the 15×15 SCC CNN's with the periodic boundary condition and choose, in equations (66)–(69), a parameter group$\{\alpha,\beta\}=\{10,15\}$. Firstly, let us simulate the dynamic patterns of the SCC Equations. Our simulation results are listed in **table 1**. The numerical simulation have shown that the qualitative behaviors of the SCC Equations (62)–(65) and the SCC CNN's are similar if the diffusion parameters D_1 and D_2 are small enough. Roughly speaking, if the parameters a and b are selected in the edge of chaos (black) of the equilibrium Q_2 shown in the top in figure 1(b), the dynamic behaviors of the corresponding SCC CNN's might be oscillation (see table 1). If the parameters a and b are chosen in the inner region of the locally active domain (gray) shown figure 1(b), the corresponding SCC CNN's may exhibit successively oscillatory, divergent, chaotic and convergent patterns. For most cases if the parameters a and b are located in the locally active doain which are nearby the bottom edge of chaos shown in figure 1(b), chaotic patterns can be generated.

Figures 2(a), 2(b) and 2(c) exhibit, respectively, chaotic trajectories and periodic trajectories which are generated via the SCC equations numbered by 3 and 10 listed in table 1 with different initial conditions.

Now, let us simulate their corresponding 15×15 SCC CNN's with periodic boundary conditions.

- The patterns generated by the parameter group No. 3. The parameter group is located in the locally active domain and nearby the oblique edge of chaos with respect to the equilibrium point Q_2 (see figure 1 (b)). **Figures 3**(a) and 3(b) show the trajectories of the components of the states X_1,X_2, and X_3 of the SCC CNNs are very similar to those of the original SCC equations (see figure 2(a) and (b)). The graphs of the time evolution of the patterns of the local state variables of the SCC CNN over the time interval [0, 40] are shown in **figures 4** and **5**, respectively. The diffusion parameters D_1, D_2 are selected as 0.01. The initial condition is chosen as follows. For figure 3(a) and figure 4:

$$x_{1i,j}(0)=\begin{cases}-0.1630, & \text{if } 7\le i\le 9 \text{ and } j=8 \text{ or } 7\le j\le 9 \text{ and } i=8\\ 0.1630, & \text{otherwise.}\end{cases}$$

$$x_{2i,j}(0)=\begin{cases}-0.0265, & \text{if } 7\le i\le 9 \text{ and } j=8 \text{ or } 7\le j\le 9 \text{ and } i=8\\ 0.0265, & \text{otherwise.}\end{cases}$$

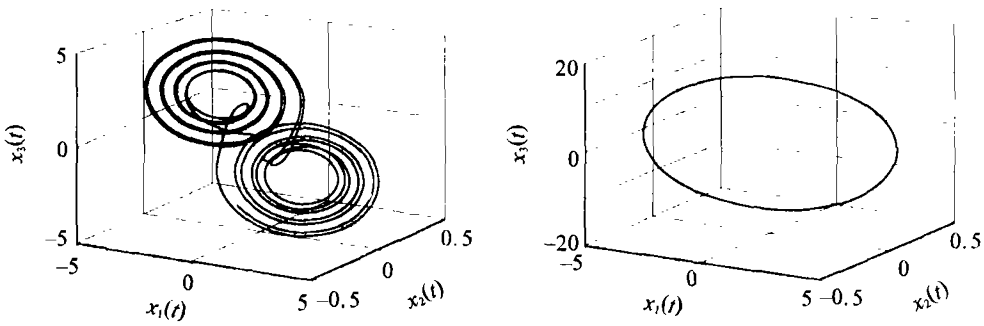
$$x_{3i,j}(0)=\begin{cases}0.0602, & \text{if } 7\le i\le 9 \text{ and } j=8 \text{ or } 7\le j\le 9 \text{ and } i=8\\ -0.0602, & \text{otherwise.}\end{cases}$$

$x_{4i,j}(0)=$

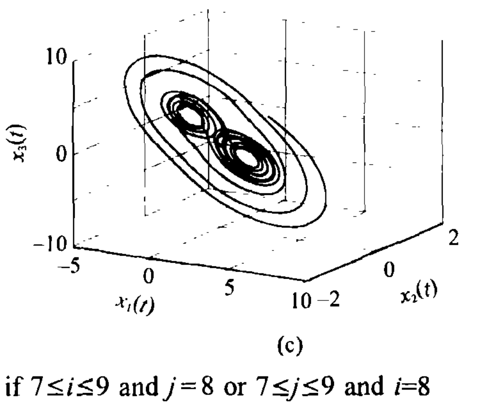
Table 1 Cell parameters, equilibrium points eigenvalues and the dynamic properties of SCC CNN's.

No.	a	b	Equilibrium point	Eigenvalue	Pattern
1*	−1	0.60	0.3239, 0, −0.3239, 0.9718	−10.0000, −0.0211±3.6814 i, −9.9299	○
2	−1	0.48	0.3239, 0, −0.3239, 0.9718	−10.0000, 0.0041±3.6572 i, −9.3073	⊕○
3	−1	0.00	0.3239, 0, −0.3239, 0.9718	−10.0000, −0.1218±3.5017 i, −6.8512	⊕○
4	−1	−0.50	0.3239, 0, −0.3239, 0.9718	−10.0000, −0.0002±3.6615 i, −9.4109	⊕⇑
5	−1	−0.85	0.3239, 0, −0.3239, 0.9718	−10.0000, −0.0936±2.4924 i, −2.0282	○⇑
6*	−1.3	−0.90	0.8639, 0, −1.8639, 5.5917	−10.0000, −0.1173±2.5340 i,−2.1622	⇓⇑
7*	−1.3	0.30	0.4523, 0, −0.4523, 1.3568	−10.0000, −0.0161±3.6768 i, −9.8026	○⇑
8	−1.3	−0.10	0.5073, 0, −0.5073, 1.5218	−10.0000, 0.0833±3.5623 i, −7.5973	○
9	−1.3	−0.85	0.1924, 0, −1.3606, 4.0819	−10.0000, 0.1924±2.7136 i, −2.7334	⇑
10	−1.3	−0.90	1.8639, 0, −1.8639, 5.5917	−10.0000, 0.1173±2.5340 i, −2.1622	⊕⇑
11	−1.3	−0.93	2.5090, 0, −2.5090, 7.5271	−10.0000, 0.0305±2.4014 i, −1.7237	○⇑
12*	−1.3	−0.94	2.8670, 0, −2.8670, 8.6010	−10.0000, −0.0111±2.3532 i, −1.5500	⇓
13*	−2	−0.94	8.7041, 0, −8.7041, 26.1124	−10.0000, −0.0018±2.3633 i, −1.5875	⇓
14	−2	−0.935	8.0630, 0, −8.0630, 24.1889	−10.0000, 0.0205±2.3892 i, −1.6806	○
15	−2	−0.80	2.8670, 0, −2.8670, 8.6010	−10.0000, 0.2332±2.9028 i, −3.3738	⊕⇑
16	−2	−0.70	2.0313, 0, −2.0313, 6.0939	−10.0000, 0.2370±3.1260 i, −4.2754	⇑
17	−2	0.00	0.8535, 0, −0.8535, 2.5604	−10.0000, 0.0024±3.6589 i, −9.3478	○
18*	−2	0.10	0.8069, 0, −0.8069, 2.4207	−10.0000, −0.0244±3.6844 i,−10.0166	○

Note: Fixed parameters $\{\alpha, \beta\} = \{10, 15\}$, $D_1 = D_2 = 0.01$ or 0.1. The symbols ⇑, ⇑, ○, and ⊕ indicate convergent, divergent, periodic and chaotic patterns, respectively. i* represents that the cell parameters lie on the edge of chaos domain.



(a) (b)



(c)

Figure 2 The chaotic trajectory and the periodic trajectory generated by the parameter set No.3 listed in table 1, (a) with the initial codition (−0.1630, −0.0265, 0.0602, −0.4275); (b) (−0.3342, 1.2220, 12.5481, −2.2370); (c) the chaotic trajectory generated by the parameter set No.10 listed in table 1 with the initial codition (1, 1, 1, 1).

$$\begin{cases} -0.4275, & \text{if } 7 \le i \le 9 \text{ and } j=8 \text{ or } 7 \le j \le 9 \text{ and } i=8 \\ 0.4275, & \text{otherwise.} \end{cases}$$

For figure 3(b) and figure 5

$x_{1i,j}(0) =$

$$\begin{cases} -0.3342, & \text{if } 7 \le i \le 9 \text{ and } j=8 \text{ or } 7 \le j \le 9 \text{ and } i=8 \\ 0.3342, & \text{otherwise.} \end{cases}$$

$x_{2i,j}(0) =$

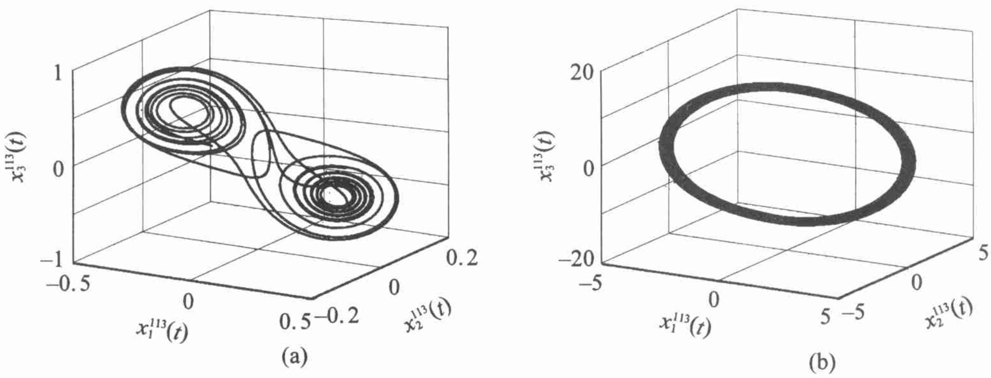
$$\begin{cases} 1.2220, & \text{if } 7 \le i \le 9 \text{ and } j=8 \text{ or } 7 \le j \le 9 \text{ and } i=8 \\ -1.2220, & \text{otherwise.} \end{cases}$$

$x_{3i,j}(0) =$

$$\begin{cases} 12.5481, & \text{if } 7 \le i \le 9 \text{ and } j=8 \text{ or } 7 \le j \le 9 \text{ and } i=8 \\ -12.5481, & \text{otherwise.} \end{cases}$$

$x_{4i,j}(0) =$

$$\begin{cases} -2.2370, & \text{if } 7 \le i \le 9 \text{ and } j=8 \text{ or } 7 \le j \le 9 \text{ and } i=8 \\ 2.2370, & \text{otherwise.} \end{cases}$$



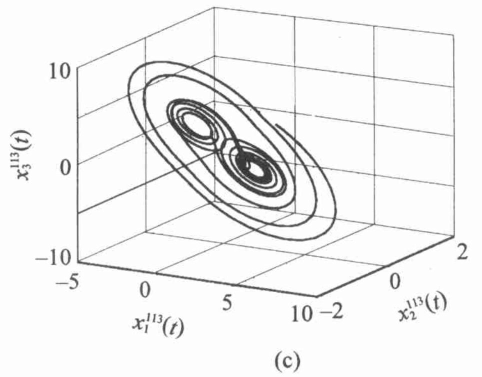


Figure 3 The trajectories of the SCC CNNS generated by the parameter set No. 3 listed in table 1 with different initial conditions, (a) the chaotic trajectory; (b) the trajectory converges to a limit cycle; (c) the chaotic trajectories generated by the parameter set No.10 listed in table 1.

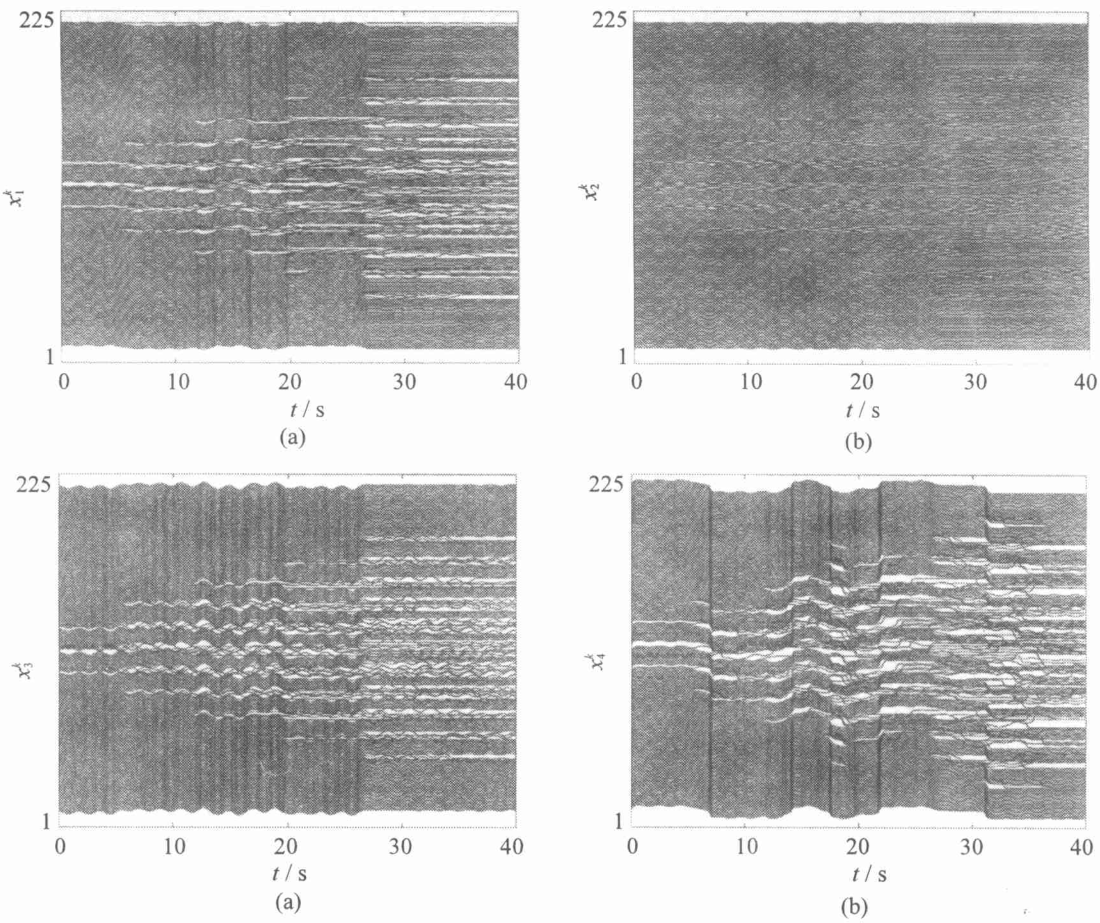


Figure 4 Graphs of the time evolution of the state variables (a) x_1^k, (b) x_2^k, (c) x_3^k, and (d) x_4^k of SCC CNNs generated by the parameter set No.3 listed in table 1 with different initial conditions.

• The patterns generated by the parameter group No. 10. The parameter group is located in the locally active domain and nearby the horizontal edge of chaos with respect to the equilibrium point Q_2 (see figure 1(b)). Figure 3(c) shows the trajectories of the components of the states $\boldsymbol{X}_1, \boldsymbol{X}_2$, and $\boldsymbol{X}_3$ of the SCC CNNs are very

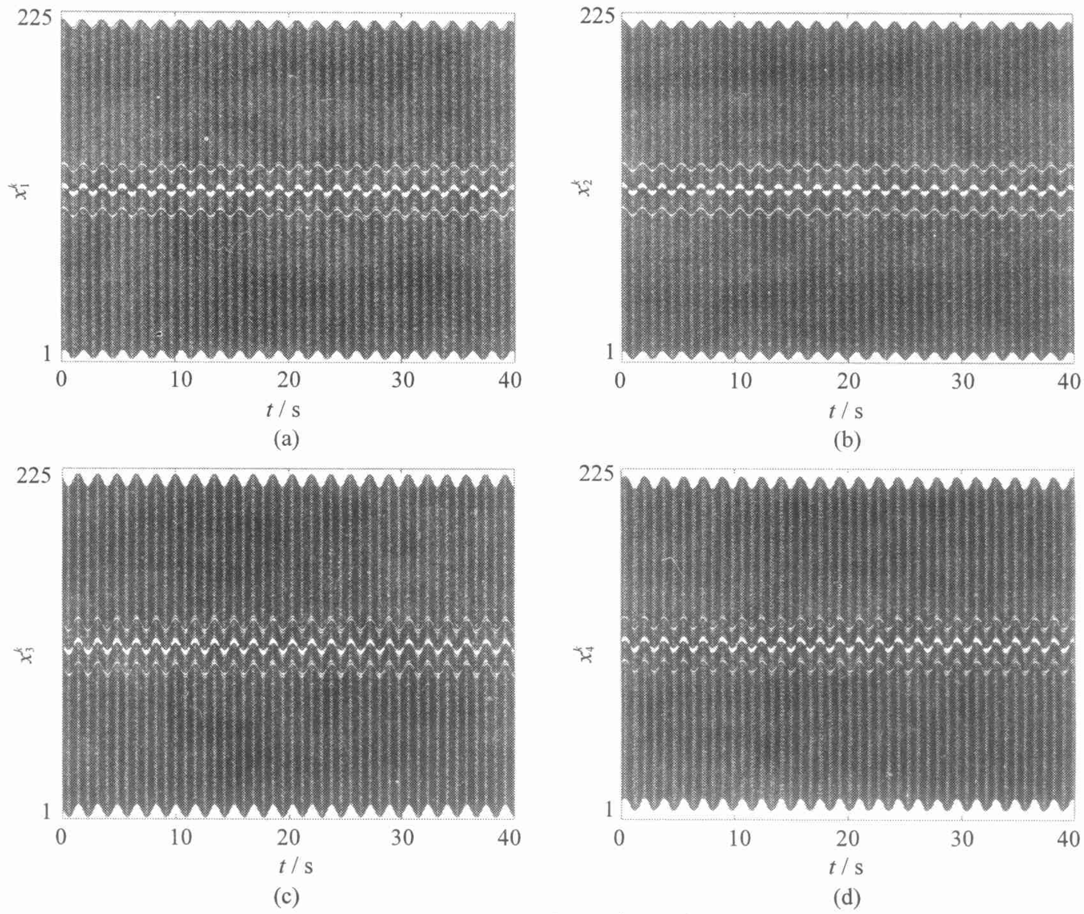


Figure 5 Graphs of the time evolution of the state variables (a) x_1^t, (b) x_2^t, (c) x_3^t, and (d) x_4^t of SCC CNNs generated by the parameter set No.3 listed in table 1.

similar to those of the original SCC equations (see figure 2(c)). The graphs of the time evolution of the chaotic patterns of the local state variables of the SCC CNN over the time interval [0, 40] are shown in **figure 6**. The diffusion coefficients D_1 ,D_2 are selected as 0.01. The initial condition is chosen as follows.

$$x_{1i,j}(0) = \begin{cases} 1, & \text{if } 7 \le i \le 9 \text{ and } j=8 \text{ or } 7 \le j \le 9 \text{ and } i=8 \\ -1, & \text{otherwise.} \end{cases}$$

$$x_{2i,j}(0) = \begin{cases} 1, & \text{if } 7 \le i \le 9 \text{ and } j=8 \text{ or } 7 \le j \le 9 \text{ and } i=8 \\ -1, & \text{otherwise.} \end{cases}$$

$$x_{3i,j}(0) = \begin{cases} 1, & \text{if } 7 \le i \le 9 \text{ and } j=8 \text{ or } 7 \le j \le 9 \text{ and } i=8 \\ -1, & \text{otherwise.} \end{cases}$$

$$x_{4i,j}(0) = \begin{cases} 1, & \text{if } 7 \le i \le 9 \textit{ and } j=8 \text{ or } 7 \le j \le 9 \text{ and } i=8 \\ -1, & \text{otherwise.} \end{cases}$$

4 Concluding Remarks

Based on the local activity priciple of CNN cells presented by Chua [17, 19], the analytical criteria of the local activity for CNN's with four state variables and two-port (two diffusion coefficients) are set up. The analytical critera consist of Theorems 1–4, which can be implemented by a computer program to produce bifurcation diagrams for general corresponding CNNs.

The SCC CNN with two ports is introduced and the bifurcation diagrams of the SCC CNN are calculated. Although the analytical criteria are much more complex than those of the CNN with one port [24, 25], the bifurcation diagrams of the SCC CNN with two ports and one port calculated via these criteria are the same.

The numerical simulation exhibit the extremely complex behaviors possible in the SCC CNN which are similar to the SCC equations if the diffusion coefficients of the SCC CNN (see equations (65)–(69)) are small enough. Chaotic patterns, oscillatory patterns and divergent patterns can be obtained if the corresponding cell parameters of the SCC CNN's are selected in the locally active and unstable domain but nearby the edge of chaos domains.

In summary this paper demonstrates once again that the local activty theory provides a practical tool for the study of the complex dynamics of some nonlinear systems. Clearly, further researches on this topic are very

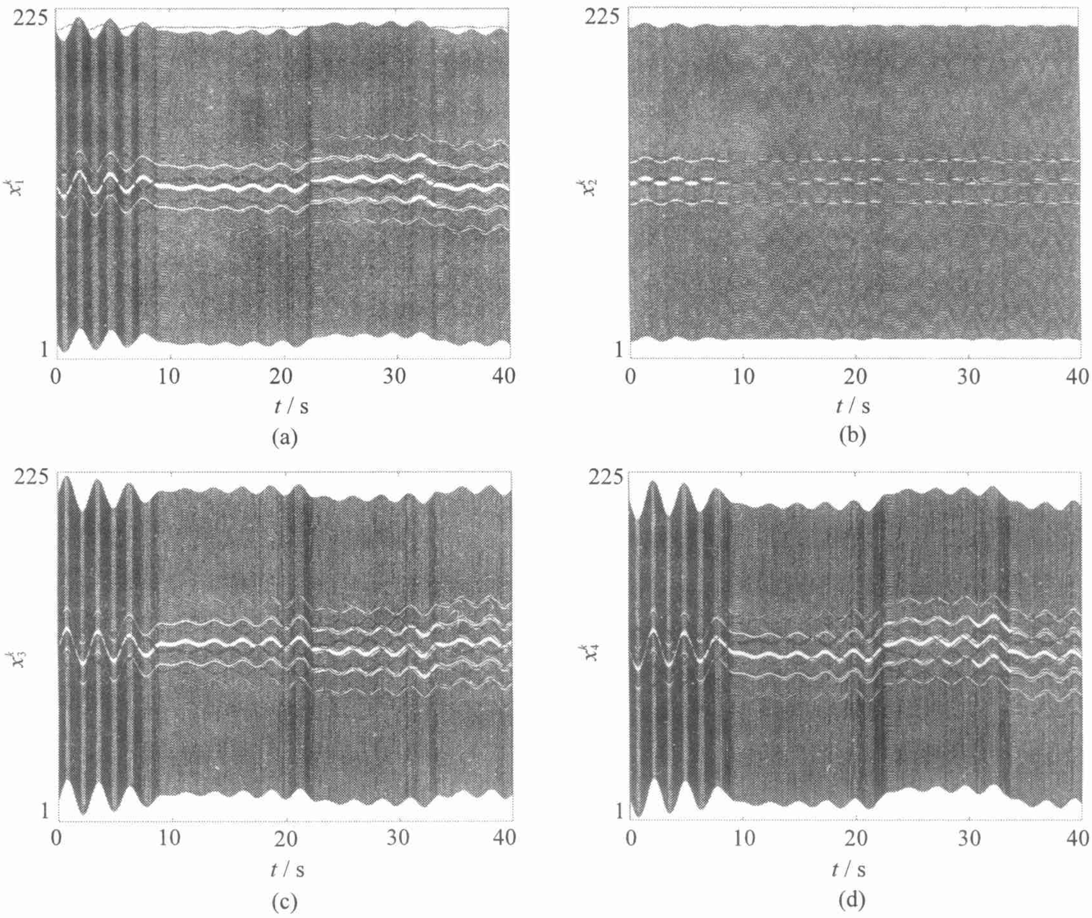


Figure 6 Graphs of the time evolution of the state variables (a) x_1^s, (b) x_2^s, (c) x_3^s, and (d) x_4^s of SCC CNNs generated by the parameter set No.10 listed in table 1.

promising and quite challenging

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