

## An Improved Minimum Distance Method Based on Artificial Neural Networks

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**Abstract:** MDM (minimum distance method) is a very popular algorithm in state recognition. But it has a presupposition, that is, the distance within one class must be shorter enough than the distance between classes. When this presupposition is not satisfied, the method is no longer valid. In order to overcome the shortcomings of MDM, an improved minimum distance method (IMDM) based on ANN (artificial neural networks) is presented. The simulation results demonstrate that IMDM has two advantages, that is, the rate of recognition is faster and the accuracy of recognition is higher compared with MDM.

**Key words:** state recognition; minimum distance method; artificial neural networks

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### 1 Introduction of Minimum Distance Method and its Weakness [1]

MDM (minimum distance method) is a popular method for state recognition. It's usually operated as following steps.

(1) Determine the central vector  $W_i$  ( $i=1, \dots, c$ ,  $c$  is the total number of classes) of class  $M_i$ .

(2) Calculate the distance  $d_i$  ( $i=1, \dots, c$ ) between the vector  $X$  and the central vector  $W_i$  ( $i=1, \dots, c$ ) by the following equation:

$$d_i = \|X - W_i\| = \left[ \sum_{k=1}^n (x_k - w_{ik})^2 \right]^{1/2} \quad (1)$$

where  $X = [x_1, x_2, \dots, x_n]$  is the vector to be recognized and  $x_k$  ( $k=1, \dots, n$ ) is the characteristic value of vector  $X$ ;  $W_i = [w_{i1}, w_{i2}, \dots, w_{in}]$  and  $w_{ik}$  ( $k=1, \dots, n$ ) is the characteristic value of the central vector  $W_i$ .

(3) Select the minimum distance  $d_j$  ( $d_j = \min_i \|X - W_i\|$ ). Then vector  $X$  is labeled as class  $j$ .

When MDM is applied to state recognition, a presupposition, that is the distance within one class must be shorter enough than the distance between classes must be satisfied. For example, in **figure 1**, the data set  $M$  can be classified to 3 classes according to criteria  $A$ , which can be denoted as  $M = \{M_1, M_2, M_3\}$ . We can also see that  $M_1$  and  $M_3$  are composed of only one part and  $M_2$  is composed of three parts. These three parts are called subclass and denoted as  $M_{21}, M_{22}$ , and  $M_{23}$ , i.e.

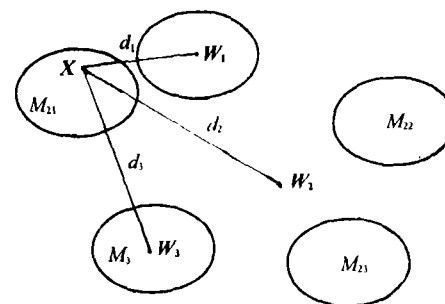


Figure 1 Sketch map of minimum distance method.

$M_2 = \{M_{21}, M_{22}, M_{23}\}$ . The subclass  $M_{21}, M_{22}$  and  $M_{23}$  can be clustered by criteria  $B$ .  $W_1, W_2$  and  $W_3$  are the central vectors of  $M_1, M_2$  and  $M_3$ . If MDM is used to determine which class the vector  $X$  belongs to, then  $d_1, d_2$  and  $d_3$  need to be computed. It is obvious that  $d_1$  is the shortest distances among  $d_1, d_2$  and  $d_3$ . So we put the vector  $X$  to class 1. In fact, vector  $X$  belongs to class 2. The recognition result is wrong.

### 2 Improved Minimum Distance Method [2]

If we classify the data set  $M$  with criteria  $C$  ( $C = A \cap B$ ),  $M$  can be grouped in 5 clusters as **figure 2** shows. In order to distinguish from the classes originally named, these 5 clusters are called new-classes and denoted as  $M'_1, M'_2, M'_3, M'_4$  and  $M'_5$ , i.e.  $M = \{M'_1, M'_2, M'_3, M'_4, M'_5\}$ . If the new-class satisfies the presupposition, MDM can be used to classify the data and will not make any mistakes. Then we can determine the central vector  $W'_i$  ( $i=1, 2, \dots, 5$ ) of  $M'_i$  ( $i=1, 2, \dots, 5$ ) and calculate the distance  $d'_i = \|X - W'_i\|$  ( $i=1, 2, \dots, 5$ ). For example, in **figure 2**, vector  $X$  can be put to new-class 2 because  $d'_2$  is the shortest distance among  $d'_i$  ( $i=1, 2, \dots, 5$ ).

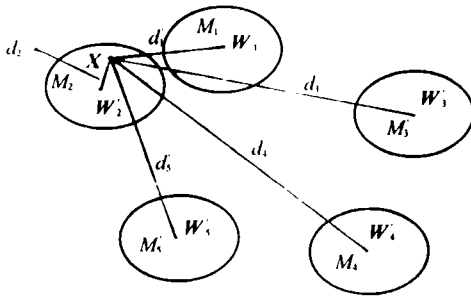


Figure 2 Sketch map of the improved minimum distance method.

Therefore we can put vector  $X$  to class 2 by use of the relationship between new-class and class as table 1 shown. We can see that the recognition result is right. We call the algorithm above IMDM (improved minimum distance method).

Table 1 Relationship between new-class and class

new-class	1	2	3	4	5
class	1	2	2	2	3

When the improved minimum distance method is used for classification, there are two problems need to be solved.

(1) How to obtain criteria  $B$  so that the new-class can satisfy the presupposition.

(2) How to determine the central vector  $W_i$  of new-class  $M_i$ .

### 3 Realization of the Improved Minimum Distance Method Based on ANN (Artificial Neural Networks)

#### 3.1 Model of neural networks [3]

The model of neural networks for state recognition is composed of two layers as figure 3 shows. In the first layer, every node is used to receive and normalize the characteristic value  $x_i (i = 0, \dots, m - 1)$  of vector  $X$ . Every node  $y_j (j = 0, \dots, n)$  of the second layer denotes a new-class. The central vector of new-class  $j$  is expressed by weight vector  $W_j = (w_{j0}, w_{j1}, \dots, w_{j, m-1})$ .

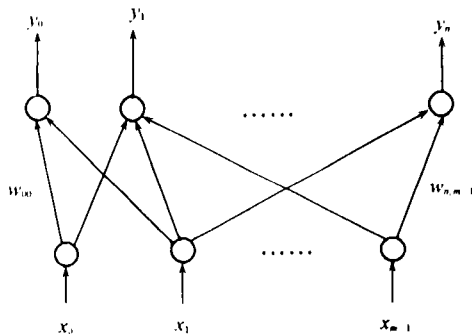


Figure 3 The structure of neural networks for state recognition.

#### 3.2 UR&SAL (unsupervised resonance & supervised adjust learning) algorithm [2, 4]

UR&SAL is a learning algorithm of neural networks. Its process can be described as follows.

Step 1 Determine the range of new-class, indicated as  $[S_s, S_b]$ . Set the bounds of distance parameter  $\rho$ , expressed as  $[\rho_s, \rho_B]$ . Normalize  $x_i (i = 0, \dots, m - 1)$  and record the total number of class as  $C$ .

Step 2 Set  $K = 1$  ( $K$  is the sequence number of class).

Step 3 Let  $L = \sum_{i=1}^K S_i$  ( $S_i$  stands for the number of subclass clustered from class  $j$  and  $L$  is the sequence number of new-class). Set  $S_k = 1$  and  $W_L = X(0)$ , where  $X(0)$  is the first input vector of class  $j$ .

Step 4 Execute the following steps to every input learning sample  $X$ .

(1) Activity. Compute the distance between  $X$  and  $W_i$  with equation (2).

$$d_i = \|X - W_i\| = \left[ \sum_{j=0}^{m-1} (x_j - w_{ij})^2 \right]^{1/2} \quad (2)$$

(2) Competition. Find vector  $W_i$  which corresponding to  $d_i = \min d_i$ . If there are many vectors, select one at random.

(3) Resonance. If  $d_i > \rho$ , then adding a new node to the second layer and give it a new sequence number  $L$ . At the same time, set  $W_i = X$ ,  $S_k = S_k + 1$ , and  $L = L + 1$  turn to step 5.

(4) Learning. If  $d_i \leq \rho$ , then set  $W_i' = W_i + \beta(X - W_i)$ , where  $\beta$  is the learning efficiency.

Step 5 If the learning samples of class  $K$  have been completely inputted, turn to step 6. Otherwise turn to step 4.

Step 6 If  $S_k \in [S_s, S_b]$  is satisfied, turn to next step. Otherwise update  $\rho$  and turn to step 3.

Step 7 If  $K \leq C$ , then set  $K = K + 1$  and turn to step 3. Otherwise turn to next step.

Step 8 Initialize the weight vector with equation  $W_L(0) = W_L (L = 1, 2, \dots, n)$ , where  $n (n = \sum_{i=1}^K S_i)$  stands for the total number of the new-class.

Step 9 Select a vector  $X(t)$  from the set of input vector randomly and compute the distance  $d_L$  between  $X(t)$  and  $W_L(t)$ . If  $d_L = \min d_L(X(t), W_L(t))$ , then  $X(t)$  should be put in the new-class  $r$ . By virtue of the relationship between the new-class and class, we can infer that  $X(t)$  belongs to class  $K$ . In fact,  $X(t)$  belonged to class  $K$  for it was a learning sample we had known.

Step 10 Update the weight vector  $W_r'(t)$  with following equation:

$$\begin{cases} W_r'(t+1) = W_r'(t) + \alpha(t)[X(t) - W_r'(t)] & K=K' \\ W_r'(t+1) = W_r'(t) - \alpha(t)[X(t) - W_r'(t)] & K \neq K' \\ W_L'(t+1) = W_L'(t) & L \neq r \end{cases} \quad (3)$$

where  $\alpha(t)$  is a step function such as  $\alpha(t) = \alpha_0(1 - t/T)$ , where  $t$  is the learning steps,  $\alpha_0$  and  $T$  are constants and can be determined by user.

Step 11 If the learning samples have been completely inputted, turn to next step. Otherwise turn to step 9.

Step 12 Calculate the error with following equation:

$$E = \max_L [0.5 \times \|W_L'(t+1) - W_L'(t)\|^2] \quad (4)$$

If the condition " $E < \varepsilon$ " is satisfied, stop the recurrent process, otherwise turn to step 9, where  $\varepsilon$  is the error bounds which given by user.

From the above process, two conclusions can be drawn:

(1) Step 1–step 7 is the first stage of the learning, which is called unsupervised resonance. It is similar to ART (adaptive resonance theory) algorithm and guarantees that the new-class satisfies the presupposition.

(2) Step 8–step 12 is the second stage of the learning, which is called supervised adjust learning. Its main purpose is to force the weight vector to approach the central vector of the new-class. At the same time it can also rectify the error of the first stage.

#### 4 Applications of the Improved Minimum Distance Method

In order to validate the feasibility of the improved minimum distance method, the method is applied to the recognition of the three-section heating furnace. After feature extraction, the feature space is composed of four variables. They are the temperature of heating section I, the temperature of heating section II, the temperature of soaking section and the pressure of furnace. Consequently there are four input nodes in this neural networks. The data on-site is classified to 5 classes denoted as  $M = \{M_1, M_2, M_3, M_4, M_5\} = \{\text{"Normal"}, \text{"High Temperature"}, \text{"Low Temperature"}, \text{"High Pressure"}, \text{"Low Pressure"}\}$ . There are 120 samples measured on-site altogether. Among them, 80 samples are treated as learning samples and 40 samples are used as testing data. The data on-site is not satisfied the presupposition discussed above. So MDM can not be utilized directly.

Here the improved minimum distance method (IMDM) is used to this recognition process. Set  $[S_s, S_B] = [1, 10]$ ,  $[\rho_s, \rho_B] = [0.001, 0.005]$  and  $\rho = 0.002$  because

all the vectors have been normalized. After the implementation of step 1–step 7, it can be seen that  $M_1$ ,  $M_4$  and  $M_5$  are composed of only one subclass, however  $M_2$  and  $M_3$  are composed of seven subclasses. For example,  $M_2 = \{M_{21}, M_{22}, M_{23}, M_{24}, M_{25}, M_{26}, M_{27}\} = \{\text{"Only the temperature of soaking section is high"}, \text{"Only the temperature of heating section I is high"}, \text{"Only the temperature of heating section II is high"}, \text{"Both the temperatures of soaking section and heating section I are high"}, \text{"Both the temperatures of soaking section and heating section II are high"}, \text{"Both the temperatures of heating section I and heating section II are high"}, \text{"the temperatures of soaking section, heating section I and heating section II are all high"}\}$ , and  $M_3$  is similar to  $M_2$  except it corresponds to the "Low Temperature". The clustering result shows that the first stage of UR&SAL algorithm is absolutely reasonable. So there are 17 ( $=1+7+7+1+1$ ) output nodes in the second layer. In succession, we can determine the central vector  $W_i'$  ( $i=1, \dots, 17$ ) by use of step 8–step 12 of UR&SAL algorithm. The relationship between new-class and class in state recognition of heating furnace is listed in table 2.

Table 2 Relationship between new-class and class in state recognition of heating furnace

new-class	1	2–8	15	16	17
class	1	2	3	4	5
result	N (Normal)	HT(High tempera- ture)	LT(Low tempera- ture)	HP(High pressure)	LP(Low pressure)

Then the neural networks can be utilized to state recognition of heating furnace. 40 testing samples and their recognition result are listed in Appendix 1.

#### 5 Conclusions

From Appendix 1, it can be seen that the accuracy of recognition is up to 97.5%. When MDM is applied to recognition, the accuracy of recognition is only 35%. So we can say that the recognition accuracy of IMDM is higher enough than MDM. The recognition rate of IMDM is also faster than MDM because the former is executed by neural networks. In a word, the improved minimum distance method is very effective.

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Appendix 1 The recognition result of 40 testing samples

No.	Soaking/°C	Heating I /°C	Heating II/°C	Pressure/Pa	New-class	Class	Result	Correctness
1	1250	1300	1250	19.6	1	1	N	△
2	1243	1200	1157	19.6	15	3	LT	△
3	1281	1325	1263	22.6	1	1	N	△
4	1309	1354	1286	24.5	1	1	N	△
5	1283	1322	1266	16.7	17	5	LP	△
6	1267	1315	1266	23.5	1	1	N	△
7	1352	1396	1357	21.6	8	2	HT	△
8	1246	1294	1258	23.5	1	1	N	△
9	1272	1335	1269	22.6	1	1	N	△
10	1306	1355	1283	27.5	16	4	HP	△
11	1251	1305	1259	22.6	1	1	N	△
12	1248	1314	1265	21.6	1	1	N	△
13	1165	1310	1260	24.5	9	3	LT	△
14	1286	1320	1260	29.4	16	4	HP	△
15	1290	1311	1273	23.5	1	1	N	△
16	1278	1327	1265	22.6	1	1	N	△
17	1283	1330	1284	18.6	17	5	LP	△
18	1258	1306	1263	21.6	1	1	N	△
19	1275	1315	1255	22.6	1	1	N	△
20	1245	1320	1270	21.6	1	1	N	△
21	1320	1336	1304	22.6	6	2	HT	△
22	1258	1300	1254	21.6	1	1	N	△
23	1270	1285	1239	24.5	1	1	N	△
24	1257	1287	1274	20.6	1	1	N	▲
25	1256	1322	1258	23.5	1	1	N	△
26	1275	1325	1265	21.6	1	1	N	△
27	1220	1280	1270	21.6	12	3	LT	△
28	1265	1330	1265	20.6	1	1	N	△
29	1270	1329	1278	21.6	1	1	N	△
30	1250	1300	1250	17.7	17	5	LP	△
31	1259	1299	1255	22.6	1	1	N	△
32	1258	1308	1265	21.6	1	1	N	△
33	1239	1325	1180	23.5	13	3	LT	△
34	1255	1305	1255	20.6	1	1	N	△
35	1279	1328	1245	23.5	1	1	N	△
36	1264	1309	1255	21.6	1	1	N	△
37	1270	1335	1276	14.7	17	5	LP	△
38	1248	1315	1275	22.6	1	1	N	△
39	1250	1306	1254	22.6	1	1	N	△
40	1243	1320	1267	20.6	1	1	N	△

Note: △ indicates that the recognition result is right and ▲ indicates that the recognition result is wrong.