Journal of University of Science and Technology Beijing
Volume 9, Number 2, April 2002, Page 145

Information

Fuzzy optimization of space frame

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(Received 2001-04-13)

Abstract: Fuzzy concepts are introduced into structural optimization to solve fuzzy optimization problems with a crisp objective function and fuzzy constraints, also a non-membership function is used to convert fuzzy constrains into crisp constrains. Two models are discussed where the objective function considered is the volume of space frame and the fuzzy constrains are design limits by the axial strength, slenderness, deflection, thickness and diameter of space frame member.

Key words: space frame; fuzzy optimization; geometric non-linearity

[This work was financially supported by the National Natural Science Foundation of China (No.50078004).]

Steel space frame and network dome attract structural engineers because they are constructed with light-mass materials and it is relatively more flexible to design the configuration of structures [1-2]. The objective of this paper is to develop a program for stress analysis considering geometric non-linearity and to find the fuzzy optimum volume of space frame by GINO programming [3].

1 Stiffness matrix of elements

Nonlinear stiffness equations considering geometric non-linearity is derived in this paper. The strain-displacement equation could be written as the following equation, where include the second degree terms in order to consider geometric non-linearity [4]:

$$\varepsilon_x = \boldsymbol{A}_l \boldsymbol{d} + \frac{1}{2}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{d} \tag{1}$$

where,

$$[A_l] = [N_i \quad 0 \quad 0 \quad N_j \quad 0 \quad 0],$$

$$\{d\} = [d_{xi} \quad d_{yi} \quad d_{zi} \quad d_{xj} \quad d_{yj} \quad d_{zj}]^{\mathrm{T}},$$

$$[B] = \begin{bmatrix} N_i & 0 & 0 & N_j & 0 & 0 \\ 0 & N_i & 0 & 0 & N_j & 0 \\ 0 & 0 & N_i & 0 & 0 & N_j \end{bmatrix},$$

$$N_i = 1-\xi, N_j = \xi, \xi = \frac{x-x_i}{x_i - x_j}.$$

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It is assumed that the present state is initial state, and if the principle of virtual work on increment is applied to initial state, equilibrium state could be written as the following equation in the process of incremental step:

$$\int_V [(\sigma_x^{(0)} + \sigma_x)\delta\varepsilon_x]\mathrm{d}V = (\boldsymbol{f}^{(0)} + \boldsymbol{f})^{\mathrm{T}}\delta\boldsymbol{d} \tag{2}$$

In equation (2) the integral area $\mathrm{d}V$ is replaced by the sectional area A and length l. Substituting $\delta\varepsilon_x$ into equation (2), we could obtain

$$Al[(\sigma_x^{(0)} + \sigma_x)(\boldsymbol{A}_l + \boldsymbol{d}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B})] = (\boldsymbol{f}^{(0)} + \boldsymbol{f})^{\mathrm{T}} \tag{3}$$

Substituting $\sigma_x = E\varepsilon_x$ into equation (3), the following could be obtained:

$$\boldsymbol{f}^{(0)} + \boldsymbol{f} = Al(\boldsymbol{A}_l^{\mathrm{T}}\sigma_x^{(0)}) + Al[\sigma_x^{(0)}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}]\boldsymbol{d} +$$
$$AlE[\boldsymbol{A}_l^{\mathrm{T}}\boldsymbol{A}_l]\boldsymbol{d} \tag{4}$$

In equation (4), higher terms are neglected. Using the residual force γ, we could obtain

$$\boldsymbol{f} - \gamma = [\boldsymbol{k}_{\mathrm{E}} + \boldsymbol{k}_{\mathrm{G}}]\boldsymbol{d} \tag{5}$$

where,

$$\gamma = Al \cdot \boldsymbol{A}_l^{\mathrm{T}}\sigma_x^{(0)} - \boldsymbol{f}^{(0)}, \boldsymbol{k}_{\mathrm{E}} = AlE[\boldsymbol{A}_l^{\mathrm{T}}\boldsymbol{A}_l],$$

$$\boldsymbol{k}_{\mathrm{G}} = Al[\sigma_x^{(0)}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}].$$

Equation (5) is a linearized nonlinear incremental equation of space frame elements. In the equation, $\boldsymbol{k}_{\mathrm{E}}$ and $\boldsymbol{k}_{\mathrm{G}}$ are the elastic stiffness matrix and the geometric stiffness matrix of local coordinate respectively.

2 Fuzzy optimization

The constraints of ordinary optimum design of structures are rather unreasonable. For example, for a certain steel element $\sigma^{u} = 240\,\text{MPa}$ means that $\sigma = 240\,\text{MPa}$ is allowable but $\sigma = 241\,\text{MPa}$ is unacceptable. However, there is no substantive difference between $\sigma = 240\,\text{MPa}$ and $\sigma = 241\,\text{MPa}$. It is more reasonable that there should be transitional stages from absolute permission to absolute impermission when the allowable interval of a physical variable is determined. Therefore, fuzzy concepts mentioned above are introduced into structural optimization to solve fuzzy optimization problems with a crisp objective function and fuzzy constraints. A fuzzy optimization problem can be represented as follows.

(1) Model 1.

Minimize: $F(x)$ (6)

$$\text{Subject to: } \begin{cases} G_i(x) \lesssim g_i, & i=1,2,\cdots,p \\ G_j(x) \gtrsim g_j, & j=p+1,p+2,\cdots,q \\ G_k(x) \cong g_k & k=q+1,q+2,\cdots,r \\ x \geq 0 \end{cases} \tag{7}$$

where the symbol ~ means a fuzzy constraint, *e.g.*, $G \lesssim g$ means that G is approximately less than g. To solve the above fuzzy optimization problem, a non-membership function is introduced to convert fuzzy constraints into crisp constraints [5]. The non-membership function, denoted by the symbol ν, can be defined as $\nu_i(x) = 1 - \mu_i(x)$, where $\mu_i(x)$ is the membership function, while $\mu_i(x)$ represents the degree of satisfaction of $G_i(x)$ to fuzzy constraints. The membership function was introduced by Zadeh in the fuzzy set theory, and defined the membership function as [6-7]

$$\mu_i(x) = \begin{cases} 1, & \text{for } G_i(x) \leq g_i \\ 1 - \dfrac{G_i(x) - g_i}{d_i}, & \text{for } g_i < G_i(x) < g_i + d_i \\ 1, & \text{for } g_i + d_i \leq G_i(x) \end{cases} \tag{8}$$

where g_i is the allowable upper limit of the ith constraint, d_i is the tolerance of the ith constraint which is a subjectively chosen constant of admissible violation. From the relation $\nu_i(x) = 1 - \mu_i(x)$, equation (8) can be rewritten as

$$\nu_i(x) = \begin{cases} 0, & \text{for } G_i(x) \leq g_i \\ \dfrac{G_i(x) - g_i}{d_i}, & \text{for } g_i < G_i(x) < g_i + d_i \\ 1, & \text{for } g_i + d_i \leq G_i(x) \end{cases} \tag{9}$$

where $\nu_i(x)$ is the non-membership function of $G_i(x) \lesssim g_i$.

Note that the worst and best solutions are founded when $\nu = 0$ and $\nu = 1$ respectively.

Similarly, the non-membership function $\nu_j(x)$ and $\nu_k(x)$ are obtained as

$$\nu_j(x) = \begin{cases} 0, & \text{for } g_j \leq G_j(x) \\ \dfrac{g_j - G_j(x)}{d_j}, & \text{for } g_j - d_j < G_j(x) < g_j \\ 1, & \text{for } G_j(x) \leq g_j - d_j \end{cases} \tag{10}$$

$$\nu_k(x) = \begin{cases} 1, & \text{for } g_k + d_k \leq G_k(x) \\ \dfrac{G_k(x) - g_k}{d_k}, & \text{for } g_k < G_k(x) < g_k + d_k \\ 0, & \text{for } G_k(x) = g_k \\ \dfrac{g_k - G_k(x)}{d_k}, & \text{for } g_k - d_k < G_k(x) < g_k \\ 1, & \text{for } G_k(x) \leq g_k - d_k \end{cases} \tag{11}$$

From equations (9)-(11), according to Jung and Pulmano [5], fuzzy constrains in equation (7) are converted into crisp constraints as

$$\begin{cases} G_i(x) - d_i \nu \leq g_i \\ G_j(x) + d_j \nu \geq g_j \\ G_k(x) - d_k \nu \leq g_k \\ G_k(x) + d_k \nu \geq g_k \\ x \geq 0 \end{cases} \tag{12}$$

where ν is the maximal value of non-membership function $\nu_i(x)$, $\nu_j(x)$ and $\nu_k(x)$. So, Model 1 can be converted into Model 2 as follows.

(2) Model 2.

Minimize: $F(x)$ (13)

Subject to:

$$\begin{cases} G_i(x) - d_i \nu \leq g_i, & i=1,2,\cdots,p \\ G_j(x) + d_j \nu \geq g_j, & j=p+1,p+2,\cdots,q \\ G_k(x) - d_k \nu \leq g_k & k=q+1,q+2,\cdots,r \\ G_k(x) + d_k \nu \geq g_k, & k=q+1,q+2,\cdots,r \\ x \geq 0 \end{cases} \tag{14}$$

This means that solving Model 1 is equivalent to solving Model 2 which is in the form of a conventional crisp objective function and non-fuzzy constraints. Thus, conventional mathematical programming methods can solve a fuzzy optimization problem. The objective function, in this paper, is the volume of space frame and the fuzzy constraints are fuzzy design limits defined by the axial strength, maximal slenderness, minimum thickness, allowable deflection added tolerance, and ratio of outside diameter to thickness of the circular tube bar.

3 Numerical examples

Example 1. Take the case of steel latticed dome [8]. All joints of the latticed dome are located on the surface of a sphere shown as **figure 1**. We consider the dome which consists of 19 joints and 42 circular steel tubes, and the nodal force $P = 32$ kN loaded on the top joint. The internal force of the dome evaluated from geometric nonlinear analysis and all bars are of the same material with $E = 210$ GPa, $f_y = 240$ MPa. The

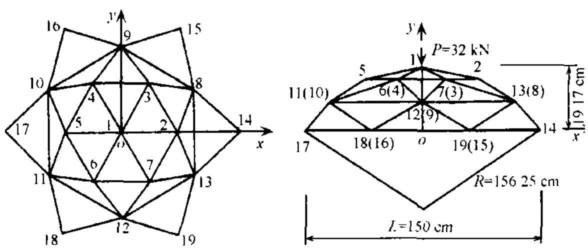

Figure 1 Steel latticed dome.

objective function and constraints can be expressed as

Minimize: $$V = \pi \sum_{i=1}^{42} (D_i - t_i) t_i l_i \tag{15}$$

Subject to: $$\begin{cases} \dfrac{N_i}{\pi(D_i - t_i)t_i} - d_f v \le f_t (\text{or } f_c) \\ \Delta - d_\Delta v \le \dfrac{L}{3} \\ \lambda_i \le 2.5 \\ \dfrac{D_i}{t_i} \le 100 \\ D_i \ge 2.72\times10^{-3}, t_i \ge 2\times10^{-4} \\ 0 \le v \le 1 \end{cases} \tag{16}$$

where, f_t, f_c are the allowable tensile stress and allowable compression stress respectively; N_i, l_i are the axial force and length of the ith circular tube respectively; d_f, d_Δ are the tolerance of permissible stress and deflection respectively; D_i, t_i are the outside diameter and thickness of the ith circular tube bar respectively.

Figure 2 shows the results of optimum design of the steel latticed dome.

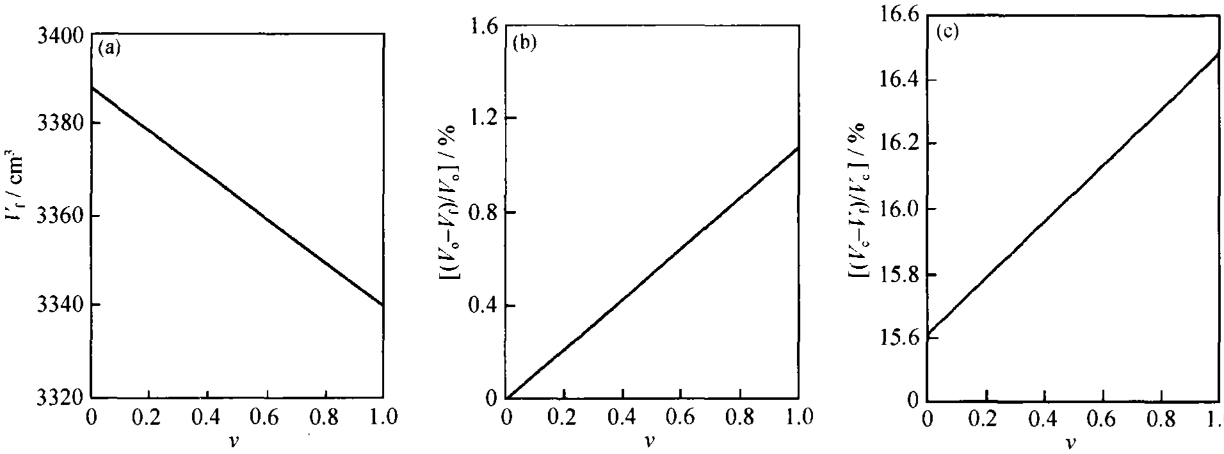

Figure 2 Fuzzy optimum design results of steel latticed dome, where V_f is the fuzzy optimum design volume, V_o is the ordinary optimum design volume, and V_c is the conventional design volume.

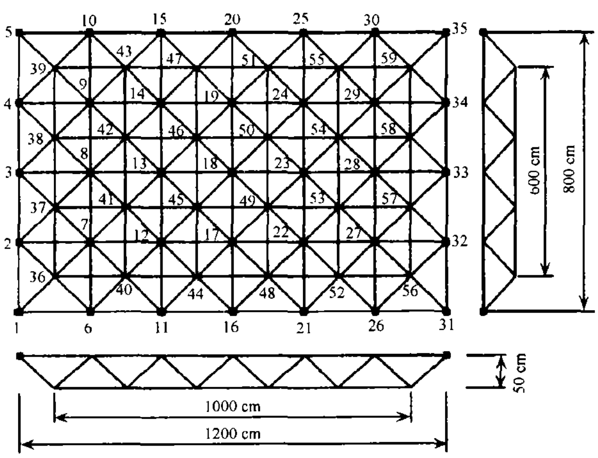

Figure 3 Double layer plan roof space truss.

Example 2. The geometry of the space frame is shown in **figure 3**, and the vertical loads applied on the joints are uniform and expressed as $P = 1$ kN/m^2. The double layer plan roof truss structure presents 59 joints and 192 circular steel tubes and it is supported around the boundary. The other design conditions are equal to the example 1. **Figure 4** shows the results of fuzzy optimum design of the double layer plan roof space truss.

4 Conclusion

Fuzzy optimization of engineering structures is considered by using a non-membership function. To formulate a fuzzy structural optimization design problem, the non-membership function is utilized in converting fuzzy constraints into scrip constraints so that conventional mathematical programming methods can solve the problem. The performance of fuzzy structural optimization is demonstrated with the solutions of two numerical examples and the fuzzy optimum area of the circular steel tubes of space frame is governed by deflection and tensile (or compression) stress.

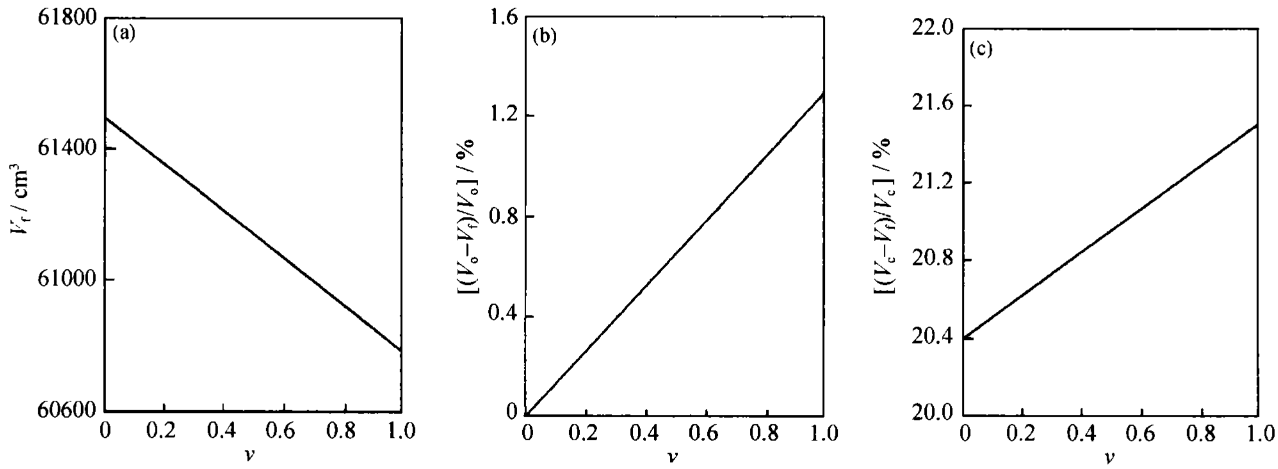

Figure 4 Fuzzy optimum design results of double layer plan roof space truss, where V_f is the fuzzy optimum design volume, V_0 is the ordinary optimum design volume, and V_c is the conventional design volume.

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