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A phenomenological constitutive equation for Rene 95 PM alloy and its application to isothermal forging process of turbine disk

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Abstract: The flow behavior of Rene 95 PM alloy was studied from 1 050 to 1 150 ℃with strain rate of 1×10^{-3}, 1×10^{-2}, 1×10^{-1} and 1 s^{-1}. At a given temperature and strain rate, flow curves exhibit a peak followed by flow softening up to a steady state. Moreover, at constant strain, flow stress increases with increasing strain rate and decreasing temperature. An equation relating hyperbolic sine of flow stress to hot working parameters, such as strain, strain rate and temperature, was established by using multiple nonlinear regression method. A very good agreement was found between predicted and experimental flow stress in all the strain range investigated. Application of the constitutive equation in predicting forming loads and flow behavior and temperature distribution in both upper and lower dies in an isothermal forging process of turbine disk of large dimension (about 630 mm) by means of a finite element code was systematically analyzed.

Key words: constitutive equation; finite element method; isothermal forging; Rene 95 PM alloy

1 Introduction

In metal-forming operations, the analytical approach to process planning for the definition of working conditions giving defect-free items with controlled dimensions, grain structure, and plastic flow so that the formed components have good mechanical properties, is becoming more useful than the trial and error one. Development of powerful process modeling techniques, such as finite element method (FEM), makes accurate simulation of large deformations during forming operations possible [1-2]. Effective application of such simulative techniques requires a precise knowledge of (1) thermo-physical properties, such as specific heat capacity and thermal conductivity of workpiece and dies, (2) die-workpiece interface data, such as frictional conditions and coefficient of heat transfer, and (3) constitutive equation of the material under deformation in order to predict the flow stress of the workpiece under continuously changing conditions of strain, strain rate, and temperature, even microstructure parameters, if necessary. Several studies have been undertaken in these fields [3-4]. In particular, a lot of constitutive equations, describing the nonlinear relationship existing among working parameters, such as stress, strain, strain rate, and temperature, were proposed for several materials [5-6]. However, those materials like nickel-based PM superalloys and wrought powder metallurgy (PM) superalloys have not been extensively investigated. Availability of experimental data and constitutive equations of plastic flow for such alloys is extremely important since they are hot formed into a final shape like conventional ingot casting alloys. By now, processing design of isothermal forging for PM superalloy components such as turbine disks of large dimensions is still a difficult problem in China.

In this paper, flow behavior of PM Rene 95 alloy at high temperatures was systematically studied. Constitutive equations of flow stress versus hot working parameters such as strain, strain rate and temperature were established using multiple nonlinear regression method. Such equations have been successfully employed in a FEM code-FORMT (forming with coupled temperature) to simulate isothermal forging process for PM Rene 95 alloy disk of large dimensions.

2 Material and experimental procedure

2.1 Material

The Rene 95 PM superalloy was supplied by Steel & Iron Research Institute, Beijing, China. It was produced by hipping powders into billets of dimensions of 76 mm in diameter and 102 mm in height. The billets were hipped at 1 150 ℃, 120 MPa for 3 h. Then they were machined into the compression specimens of 8 mm in diameter and 12 mm in height. The chemical composition of Rene 95 PM alloy is as follows (mass fraction in %): C, 0.051; Al, 3.55; W, 3.65; Nb, 3.50; Mo, 3.62; Ti, 2.62; Cr, 13.01; Co, 8.04; Fe, <0.10; B, 0.003 3; Mn, 0.025; Zr, 0.047; Si, 0.11; P, <0.005; S, 0.001; H, 2.63

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$\times 10^{-6}$; N, 1.4×10^{-5}; O, 3.9×10^{-5}; and Ni balance.

2.2 Experimental procedure

Hot compression tests were conducted on THEMECMASTER-Z computerized servo hydraulic machine. Tests were carried out in an induction-heating furnace; samples were heated to deformation temperature and held for 240 s to eliminate thermal gradients, and finally deformed at selected constant strain rate (**figure 1**). Output of the load and displacement were converted into true stress and true strain data automatically by special program. To minimize friction effect on workpiece-die interface, forging experiments were performed using FR5 as high temperature lubricant.

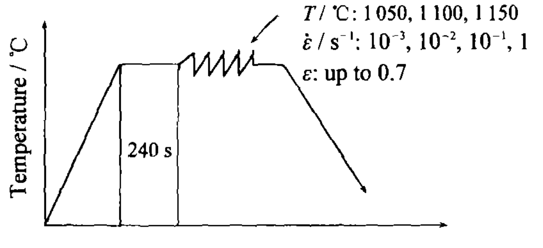


Figure 1 Schematic representation of the method used for hot compression tests.

3 Experimental results and constitutive equation

3.1 Experimental results

Typical true stress/true strain curves are presented in **figure 2** for as-hipped PM Rene 95 alloy. It can be seen that the shape of the flow curves are classical in nature, with work hardening before a given strain and the strain of the peak increasing with decreasing temperature and increasing strain rate. It is typical of materials that recrystallize dynamically when deformed at temperature above half their absolute melting temperatures.

3.2 Constitutive equation

By now, several types of constitutive models have been put forward. The most used constitutive equation to model the hot working behavior of metallic materials, proposed by Sellars and Tegart [7], correlates the equivalent flow stress σ with the equivalent strain rate $\dot{\varepsilon}$ and with temperature T as following:

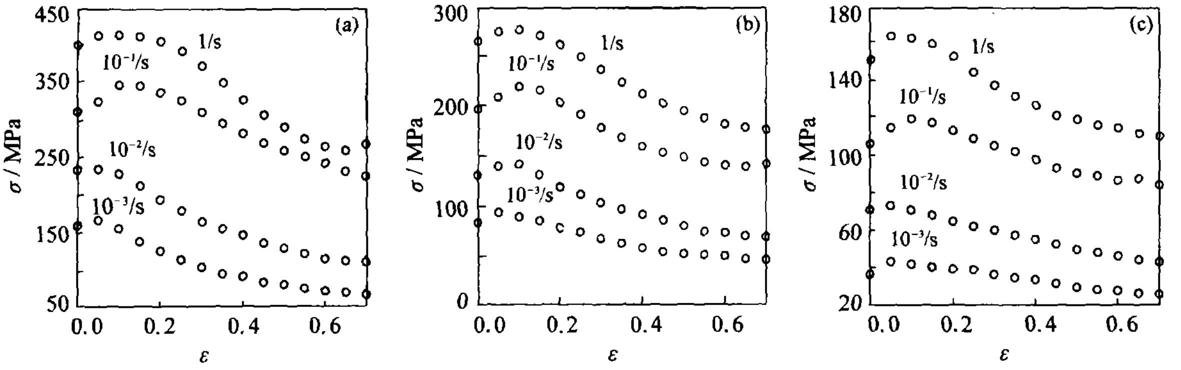


Figure 2 Experimental data for as-hipped Rene 95 alloy with varying strain rate and temperature, (a) 1 050 ℃; (b) 1 100 ℃; (c) 1 150 ℃.

$$Z=\dot{\varepsilon}\exp(Q/RT)=A[\sinh(\alpha\sigma)]^{n} \qquad (1)$$

where Z is the Zener-Hollomon parameter representing the temperature modified strain rate; Q the activation energy related to the deformation mechanisms taking place in the material during the process; R the gas constant; A, n, and α the material parameters. Such relationship applies a broad range in hot working reducing to a power law at low stresses and to an exponential law at high stresses. Equation (1), that is generally used only for particular points of the flow curves, for example at the peak or at the steady state regime of the σ-ε curves, has the advantage of simplicity. However, since equation (1) neglects the strain effects, it results in a very coarse approximation of the true behavior of materials.

Exploitation of the full potential of the metal forming modeling techniques based on FEM analysis requires the precise characterization of the material flow behavior, in particular, there is a demand for information on strain dependence of stress at each temperature and strain rate. To overcome such problems, Roberts [7] has proposed an equation describing the flow curves in hot working conditions:

$$\frac{\sigma}{\sigma_{p}}=[\frac{\varepsilon}{\varepsilon_{p}}\exp(1-\frac{\varepsilon}{\varepsilon_{p}})]^{C} \qquad (2)$$

where σ_p and ε_p are values of stress and strain at the peak of flow curves, respectively, and C is a constant. When flow curve exhibits a strain hardening stage followed by a steady-state plateau, the peak values of σ and ε can be replaced by σ_s and ε_s. Equation (2) describes the σ-ε behavior up to ε_p or ε_s; at higher strains, flow stress is kept constant at σ_p or σ_s, where σ_s and ε_s are values of strain and stress at a steady state plateau. For materials exhibiting a strain hardening stage up to a maximum flow stress followed by a flow softening until fracture, using this model will lead to the results that the predictions always exceed the true values of flow stress.

Take the above constitutive models into consideration, a new equation model is obtained:

$$\sigma=\frac{1}{\alpha}\sinh^{-1}\left[\dot{\varepsilon}\exp\frac{Q(\varepsilon)}{RT}/A(\varepsilon)\right]^{\frac{1}{n(\varepsilon)}}=\frac{1}{\alpha}\sinh^{-1}\left[\frac{Z(\varepsilon)}{A(\varepsilon)}\right]^{\frac{1}{n(\varepsilon)}} \quad (3)$$

This equation correlates flow stress with hot working parameters ε, $\dot{\varepsilon}$ and T, and takes Q, A, and n of equation (1) as a function of strain. Equation (3) accounts for strain rate and temperature sensitivity by means of the Zener-Hollomon parameter. Moreover, the sensitivity of A, Q, and n values to various restoration mechanisms taking place during deformation processes, permits to develop a more general procedure for calculating flow stress data.

Based on the tested data and equation (3), a phenomenological constitutive equation model for PM Rene 95 alloy was put forward:

$$\sigma=A\sinh^{-1}(BZ^{m}) \quad (4)$$

$$\begin{cases} Z=\dot{\varepsilon}\exp(Q/RT) \\ B=\exp[b_0+b_1\exp(b_2\varepsilon)] \\ Q=[c_0+c_1\exp(c_2\varepsilon)]\times10^5 \\ m=d_0+d_1\ln\varepsilon+d_2\ln^2\varepsilon \end{cases} \quad (5)$$

where A, b_i, c_i, d_i $(i=0, 1, 2)$ are unknown parameters.

According to the least-square method, the above-mentioned unknown parameters are treated as undefined vector X, and hot working parameters as the independent variable Y, then the nonlinear statistical model can be expressed as a function of X and Y:

$$F=f(X, Y)+\omega \quad (6)$$

where F is a dependent variable, ω is an error random variable subjected to normal distribution.

Provided that (x_i, f_{ie}), $i=1, 2, \cdots, n$, are n sets of independent samples, in which f_{ie} is the tested data corresponding to x_i, and f_{ie} and x_i are the components of vectors F_e and X, respectively. The optimal estimation of parameter X can be determined through minimizing the following objective function:

$$S=\sum_{i=1}^{n}(f_{ie}-f_i)^2 \quad (7)$$

where f_i are the components of vector F.

Combine equations (4), (5) with equation (7), let unknown variable $X=[A, b_i, c_i, d_i]^{\mathrm{T}}$ $(i=0, 1, 2)$, independent variable $Y=[\varepsilon, \dot{\varepsilon}, T]^{\mathrm{T}}$, and dependent variable F is the desired flow stress, while vector F_e the tested flow stress, the unknown variable in equations (4) and (5) can be obtained (see **table 1**) by using Newton-Raphson method. Values of vector X are illustrated in table 1.

The predicted flow curves from 1 050 to 1 150 ℃ with strain rate of 1×10^{-3}, 1×10^{-2}, 1×10^{-1} and $1\ \mathrm{s}^{-1}$, calculated by equation (4), are shown in **figure** 3. In general, the difference between predicted and measured flow stress values is less than 5%.

Table 1 Coefficients of as-hipped PM Rene 95 constitutive equation

A	b_0	b_1	b_2	c_0	c_1	c_2	d_0	d_1	d_2
276.395 8	10.959 3	−31.518 1	−0.426 7	8.489 8	5.547 9	−7.410 8	0.179 9	−0.041 8	−0.013 3

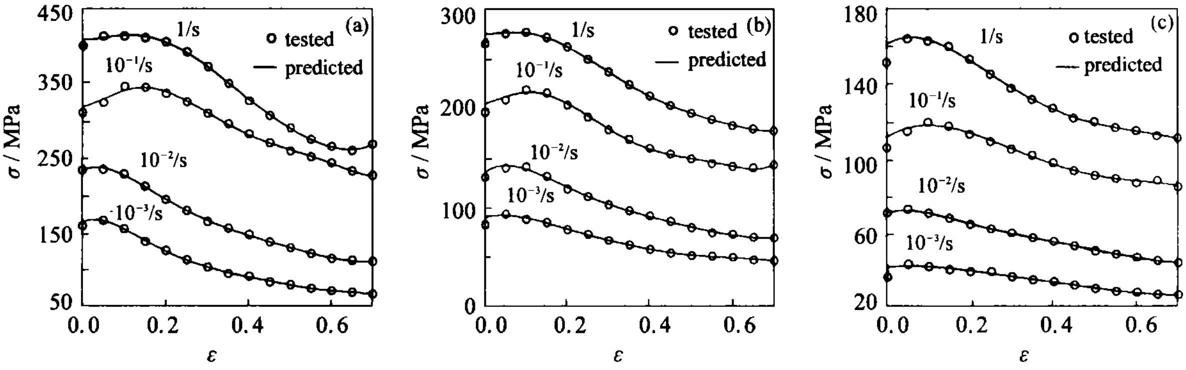


Figure 3 Predicted and tested stress response of as-hipped Rene 95 alloy, (a) 1 050 ℃; (b) 1 100 ℃; (c) 1 150 ℃.

4 Isothermal forging of PM Rene 95 turbine disk

As validation, this constitutive relationship was applied to isothermal forging process for turbine disk components of large dimension. The hot working parameter range of isothermal forging for each alloy is often determined by the conditions when super plasticity occurs. Theoretically, the criterion of super plasticity occurs when the strain rate sensitivity index *m* is greater than 0.3. It was known from reference [9] that super plastic forging parameter range for PM Rene 95 alloy is from 1 050 to 1 120 ℃ with the strain rates from 1×10^{-4}/s to 1×10^{-3}/s. Isothermal forging is somewhat different from super plastic forging in practice, the forming parameters selected for Rene 95 here are at a temperature of 1 050 ℃ and strain rate of 1×10^{-3}/s. Adopting the upper limit of strain rate range is to increase production efficiency. The above analysis shows that isothermal forging of superalloys is relative complex. The key problem of processing is how to design isothermal forging dies including how to determine the structure of the dies and how to choose die materials. For PM Rene 95 alloy, die material must have enough strength and ductility at high temperatures. By now, only one type of molybdenum-based alloy (TZM) can meet practical requirement and is widely used in other countries [10]. However, this alloy must be worked under inert-atmosphere because of its poor oxidation resistance at high temperatures.

In this section, a coupled FEM code FORMT [10] was used to simulate isothermal forging process of Rene 95 alloy turbine disk of large dimension (about 630 mm in outer diameter), the thermal-physical properties of die and materials [10] are illustrated in **table 2**. Isothermal forging process was simulated at a pseudo-constant strain rate of 1×10^{-3}/s.

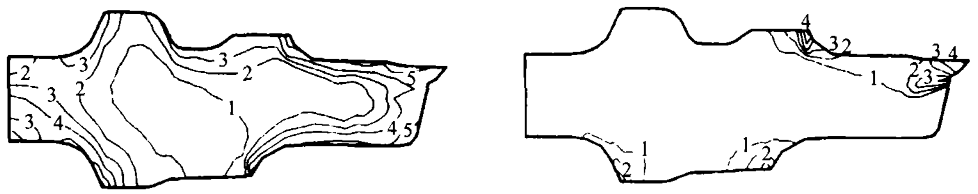
Table 2 Thermal-physical properties of die and materials

Thermal conductivity of workpiece / $(W\cdot m^{-1}\cdot K^{-1})$	$9.6 + 0.0132T$
Thermal conductivity of dies / $(W\cdot m^{-1}\cdot K^{-1})$	$138.46 - 0.0296T$
Specific heat capacity of workpiece / $(J\cdot kg^{-1}\cdot K^{-1})$	590.37
Specific heat capacity of dies / $(J\cdot kg^{-1}\cdot K^{-1})$	669.58
Density of workpiece / $(kg\cdot m^{-3})$	8 300
Density of dies / $(kg\cdot m^{-3})$	10 160
Coefficient of heat transfer for die / workpiece interface / $(W\cdot m^{-2}\cdot K^{-1})$	2 850
Coefficient of heat transfer for medium/$(W\cdot m^{-2}\cdot K^{-1})$	11.36
Radiation coefficient / $(W\cdot m^{-2}\cdot K^{-4})$	5.67×10^{-8}

5 Discussions

Although equation (4), as a multiple nonlinear function of the hot working parameters, describes the experimental data well, it only utilizes the fit property—one of the basic characteristics of nonlinear regression. Theoretically, the regression model should undergo further optimization at the given fit property. In fact, the constitutive relationships for deformed materials have two primary functions: one is describing the deformation mechanisms of material studied; the other is providing the material database for further numerical simulation of specified processes. Theoretically, an unreasonable constitutive equation model could make the FEM iteration un-convergent. The object of establishing the constitutive relationship here is mainly to simulate the hot forming processes like forging, rolling *etc*., so equation (4) has enough precision for further engineering use.

Applying this constitutive relationship to isothermal forging of turbine disk component provides satisfied results, as can be seen from the following figures. **Figure** 4(a) gives the distribution of effective strain at the end of deformation. Differing from conventional forging, such as hot die forging, the effective strain in the edge region is greater than that in the inner region of the forgings because of the thermal effect induced by deformation and friction at the die/workpiece interface. The effective strain rate distribution (figure 4(b)) shows that the majority region of the component is deformed at a rate lower than 2×10^{-3}/s, which is greater than the designed one (1×10^{-3}/s) due to shape complexity of the forgings, but the predicted result and the desired one are of the same magnitude. The key problem of isothermal forging is the temperature increase of dies, which is illustrated in **figure 5**. The highest temperatures in upper and lower die are 1 127.44 and 1 133.99 ℃,respectively. They are both within the failure temperature, about 1 700 ℃, of TZM alloy. For PM Rene 95 disk component, isothermal forging at 1 100 ℃ with a strain rate of 1×10^{-3}/s requires a forming equipment capacity of about 60 MN (**figure 6**).



(a) (b)

Figure 4 Flow behavior of as-hipped Rene 95 disk component at the end of forging, (a) effective strain distribution, max—1.691, min—0.345, 1—0.457, 2—0.682, 3—0.906, 4—1.130, 5—1.354, 6—1.579; (b) effective strain rate distribution (1/s), max—0.022 9, min—0.000 1, 1—0.002 0, 2—0.005 8, 3—0.009 6, 4—0.0134, 5—0.017 0, 6—0.021 0.

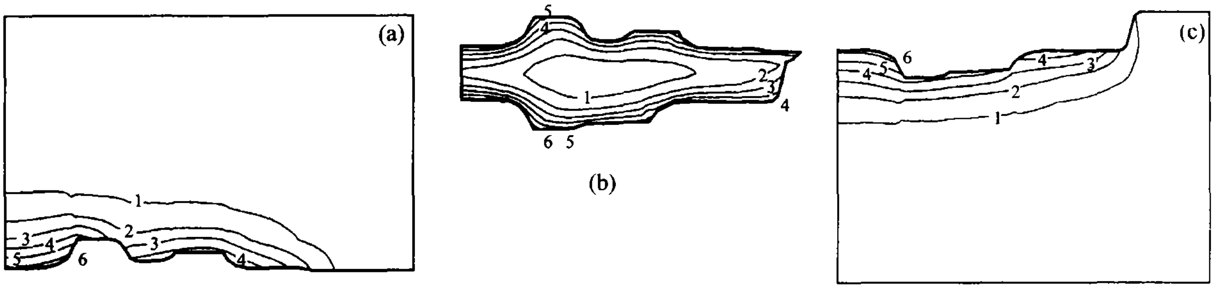


Figure 5 Temperature contours in dies and workpiece at the end of forging (℃), (a) Upper die, max—1 134.48, min—1 050.00, 1—1 057.04, 2—1 071.12, 3—1 085.20, 4—1 099.28, 5—1 136.36, 6—1 127.44; (b) Workpiece, max—1 150.70, min—1 054.27, 1—1 062.31, 2—1 078.38, 3—1 094.45, 4—1 110.52, 5—1 126.59, 6—1 142.67; (c) Lower die, max—1 141.63, min—1 050.00, 1—1 057.64, 2—1 072.91, 3—1 088.18, 4—1 103.45, 5—1 118.73, 6—1 133.99.

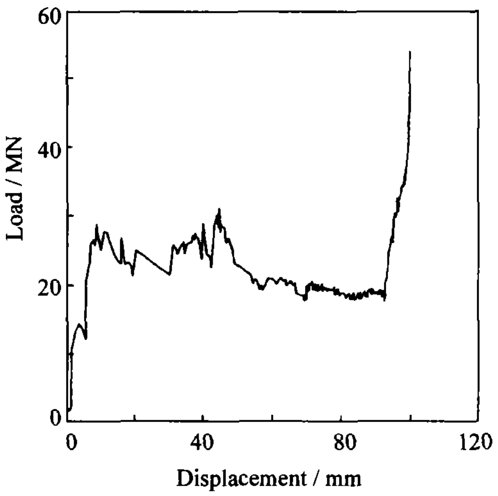


Figure 6 Load curve of closed die isothermal forging for Rene 95 turbine disk.

Processing design of actual isothermal forging and its industrial application in our country are still undergoing. The further simulation works are still being engaged in.

6 Conclusions

(1) The constitutive equation established by using multiple nonlinear regression describes flowing behavior of PM Rene 95 alloy well.

(2) The constitutive equation can be applied to FEM numerical simulation successfully, and the numerical results are applicable.

(3) Using TZM alloy as isothermal forging die material and forming at 1 050 ℃, PM Rene 95 alloy disk of large dimension (about 630 mm in outer diameter) can be worked successfully according to the predicted results.

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