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Automation

Accurate tracking control in LOM application

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Abstract: The fabrication of accurate prototype from CAD model directly in short time depends on the accurate tracking control and reference trajectory planning in (Laminated Object Manufacture) LOM application. An improvement on contour accuracy is acquired by the introduction of a tracking controller and a trajectory generation policy. A model of the *X-Y* positioning system of LOM machine is developed as the design basis of tracking controller. The ZPETC (Zero Phase Error Tracking Controller) is used to eliminate single axis following error, thus reduce the contour error. The simulation is developed on a Maltab model based on a retrofitted LOM machine and the satisfied result is acquired.

Key words: tracking control; LOM; ZPETC; trajectory planning

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The LOM technology has been widely used in industry for design conceptualization, assembly verification and simulation. It makes possible that the manufacturer can develop a prototype directly from a CAD model in a short time, which significantly reduces the time of design and market [1]. The fabrication of accurate prototype in shorter time depends on the accurate tracking control as well as the trajectory planning policy while considering acceleration capability of the LOM machine. The contours error in LOM application are essentially due to the *X-Y* positioning errors which involve the contribution of servo mechanism and the track control algorithm. The focus of this paper will be particularly oriented to tracking control and reference trajectory planning in LOM application. To evaluate the performance of the proposed approach, simulation is developed on a Maltab model based on a retrofitted LOM machine.

1 LOM Process and machine description

The LOM process is a laminated manufacturing technique, which starts by designing a boundary surface model of the part using CAD software. The CAD model is then converted into a stereolithographic (STL) file which approximates the surfaces of the model by triangular polygons. Further, the STL model is sliced into a succession of horizontal layers ordered by their coordinates in the *z*-axis. For each layer the boundaries of the slices are contour to be cutted (*i.e.* tool-path). The contour information is then downloaded to the machine.

The heart of LOM machine is the *X-Y* positioning system, which has been especially designed for high-speed and high precision laser cutting, see **figure 1**. A laser head is fixed on the slider of axis *Y*. The *X-Y* positioning system drives the laser head to cut along the contour on the paper. Once a layer is cut over, the platform moves down one thickness in the *z* direction, then another paper is put on to cut the following layer, hence, up to the top layer of the part.

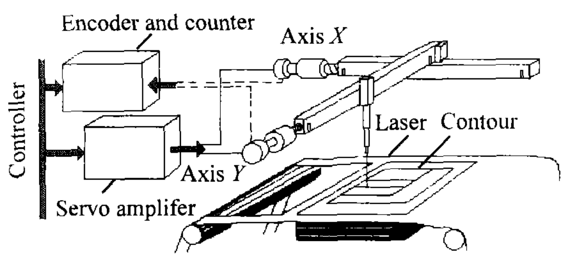

Figure 1 *X-Y* **positioning system in LOM machine.**

2 Model of *X-Y* positioning system and its controller structure

In this studied framework, the *X-Y* positioning system is a linear open-frame *x-y* table that uses high-precision ball screws and DC servomotors. A suitable preload is applied to the ball screw to keep high stiffness and no backlash. Two incremental encoder directly coupled to the servomotors are used as the velocity and

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position feedback. The dynamics equations of the DC servomotor can be reduced to equation (1)[2] .

$$\begin{cases} V_a = R_a I_a + L_a \dfrac{dI_a}{dt} + K_b \dot{\theta} \\ T = T_L + B\dot{\theta} + J_e \dfrac{d\dot{\theta}}{dt} \\ T = K_T I_a \end{cases} \tag{1}$$

Where V_a is armature voltage, V; I_a is armature current, A; R_a is winding resistance, Ω; L_a is winding inductance, mH; K_b is voltage constant, V/(rad·s^{-1}); K_T is torque constant, N·m·A^{-1}; J_e is equivalent rotor moment of inertia, kg·m^2; $\dot{\theta}$ is motor rotational speed, rad/s; B is damping, N·m/(rad·s^{-1}); and T_L is load torque which includes friction torque, N·m.

In generally, the winding inductance and the friction torque can be ignored. Therefore taking the Lapace transform of (1), the transform function of the DC servomotor is given as equation (2).

$$\frac{\theta(s)}{V(s)} = \frac{K_m}{s(T_m s+1)} \tag{2}$$

Where $K_m = \dfrac{K_T}{BR_a + K_T K_b}$, $T_m = \dfrac{J_e R_a}{(BR_a + K_T K_b)}$.

Figure 2 shows the basic schematic of the proposed tracking controller structure of *X-Y* positioning system.

A tracking feedforward compensator is added in order to compensate the regulation dynamic and impose an appropriate tracking dynamic between the new reference input $X_d(k)$ and the actual output $X(k)$. Adding feedforward compensator in axis controller can prompt the response of the single axis, consequently compensate the path following error of each individual axis. **Figure 3** shows the detail structure of the *x*-axis controller (the *y*-axis controller structure is similar).

In figure 3, A_x represents the amplifier gain, K_{vx} is the analogy velocity feedrate gain, K_{px} is the discrete position feedback gain. This discrete transfer function between the input $u_x(k)$ of the velocity loop and the position output $x(k)$ is given by

$$G_x(z) = \frac{x(z)}{u(z)} = \frac{b_0 z - b_1}{(z-1)(z-p)} \tag{3}$$

Where $b_0 = \dfrac{K_x}{a}\left(T + \dfrac{1}{a}(p-1)\right)$, $b_1 = \dfrac{K_x}{a}\left(pT + \dfrac{1}{a}(p-1)\right)$, $p = \exp(-aT)$, $a = \dfrac{1}{T_m}(A_x K_T K_{vx} + 1)$, $K_x = \dfrac{1}{2\pi T_m} A_x K_m P_1$.

The proportional and derivative gains are chosen near critically damped closed-loop behavior. The closed-loop transfer function between the desired input $x_r(k)$ and the actual position output $x(k)$ is given by

$$G_r(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = K_{px} \frac{z^{-2}(b_0 - b_1 z^{-1})}{1-(1+p)z^{-1} + (p+K_{px} b_0)z^{-2} - K_{px} b_1 z^{-3}} \tag{4}$$

$G_r(z^{-1})$ has two complex conjugate poles and one negative real pole close to the origin of the *z* plan. The feedforward compensator $T_x(z^{-1})$ compensates the close-loop transfer function $G_r(z^{-1})$ and imposes a faster tracking dynamic. However, because the zero of $G_r(z^{-1})$ ($z_1 = -0.82$) is negative, it should not be canceled, otherwise, it will result in a lightly damped oscillatory output. To ensure good tracking performance without canceling this closed-loop zero, a zero phase error tracking controller (ZPETC) was proposed by Tomizuka in reference [3] in the context of welding applications. In the ZPETC [4], the compensator $T_x(z^{-1})$ cancels the regulation dynamics defined by $A(z^{-1})$ and any stable zeros in $B(z^{-1})$. It also applies a feedforward dynamic scale factor $B(z)/[B(1)]^2$ to guarantee zero phase errors between the reference input position $x_d(k)$ and the actual position $x(k)$. In this case, the tracking compensator will be given by

$$T_x(z^{-1}) = \frac{A(z^{-1})B(z)}{[B(1)]^2} =$$

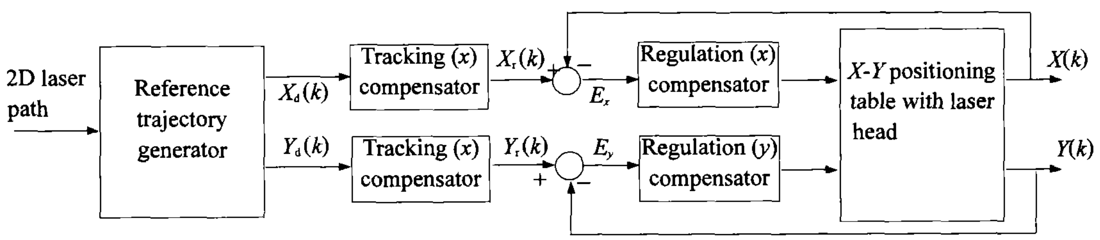

Figure 2 Proposed *X-Y* positioning system tracking controller structure.

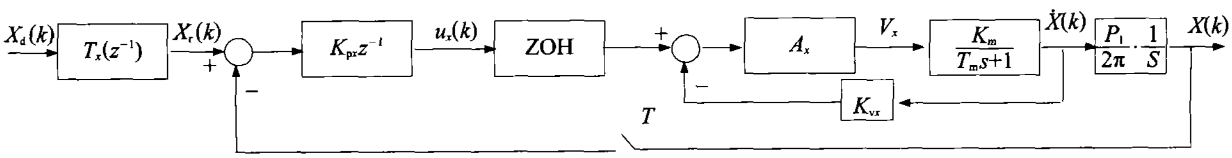

Figure 3 *X*-axis controller structure.

$$\frac{(b_0z^{-1}-b_1)(1-(1+p)z^{-1}+(p+K_{px}b_0)z^{-2}-K_{px}b_1z^{-3}}{K_{px}(b_0-b_1)^2z^{-2}} \tag{5}$$

The compensation is realizable, because the desired input sequences $x_d(k)$ and, in particular, the two-step look-ahead value, are available well in advance. Hence, at instant KT, the feedforward control action·will depend on the two-step look-ahead value of $x_d(k)$, shown by

$$x_r(k)=\frac{(b_0z^{-1}-b_1)(1-(1+p)z^{-1}+(p+K_{px}b_0)z^{-2}-K_{px}b_1z^{-3})}{K_{px}(b_0-b_1)^2}\cdot x_d(k+2) \tag{6}$$

The transfer function (7) between the reference trajectory input $x_d(k)$ and the plant output $x(k)$ expresses that the plant output is a moving average of the desired trajectory with a unite steady-state gain.

$$G_f(z)=\frac{(b_0z^{-1}-b_1)}{(b_0-b_1)}\cdot\frac{(b_0-b_1z^{-1})}{(b_0-b_1)} \tag{7}$$

3 Reference trajectory planning

Unlike the CNC in machine tool, which according to the geometrical information provided to it, uses linear or circular interpolators, in LOM application only linear interpolation is used. Linear interpolation as such contributes to the low feedrate, tracking errors, which are responsible for the precision of the final part.

Since the laser feedrate is proportional to its D/A output power that is relative to the cutting quality, it is important to keep the feedrate constant for precision contour cutting. This constitutes the first constraint for the trajectory planning policy. The second constraint is the fact that the laser feedrate is not continuous, at the connecting point between two segments, an infinite acceleration or torque is sometimes needed to exactly follow the reference contour (at sharp corners). However, since the torque is finite, the linear system model shown as equation (1) can represent a "good" approximation of the real system only when there is no saturation (*i.e.* satisfy equation (9)) [5]. The last constraint is the need to finish cutting in shorter time, which is restricted by the frequent start/stop (acceleration/deceleration) effects at the start/end point of each segment.

$$\left|J_x\frac{d\omega_x}{dt}+B\omega_x\right|\geq T_{max} \tag{9}$$

At sharp corners, the contour errors cannot be avoided unless the laser head stops at the turning points. However, it is not suitable to stop and start at each corner, especially when the segments are very small. Instead of a complete stop, it is more appropriate to reduce the speed within certain tolerances.

The trajectory generation is, therefore, an optimization problem of speed and required time while there is no jerk in motion, subject to the three constraints. From these constraints, one could conclude that the laser feedrate must be constant as much as possible, except at the sharp corners. To "optimize" the speed curve within a given line segment, it is necessary to take into account in advance the length of the next segment and the sharpness of the next corner, so that enough time is given for acceleration or deceleration.

The starting and the ending speeds of the current segment are determined according to the lengths of the current and the next segment, and according to the angle between these two segments. The flow chart in **figure 4** describes the computational steps required in this algorithm.

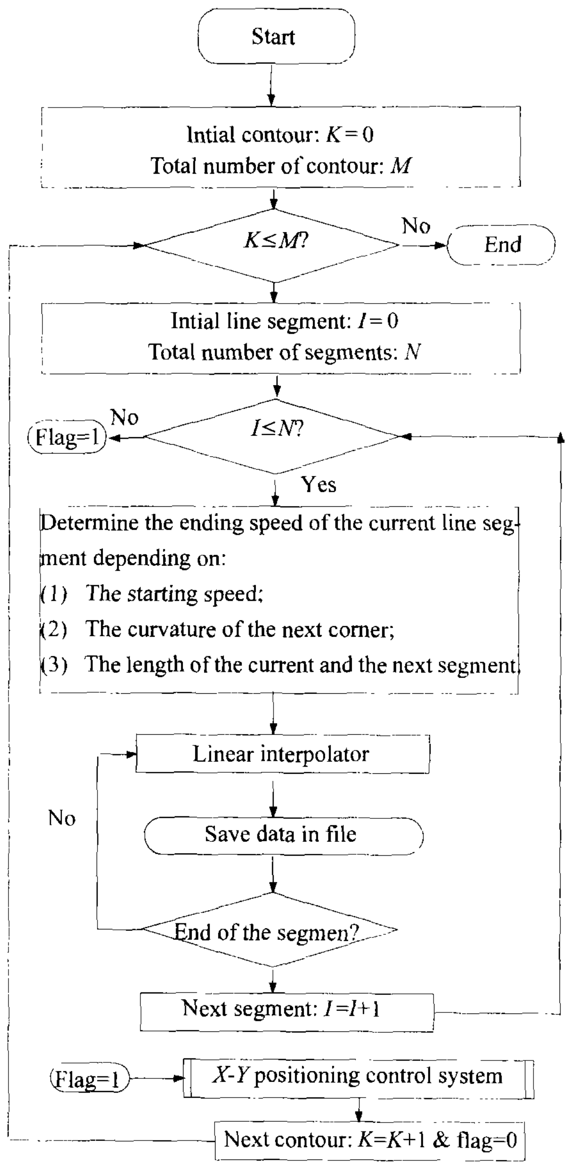

Figure 4 Flowchart of the propsed trajectory planning policy.

4 Simulation result [6,7]

A simulation model of the two-axis positioning system was developed in Matlab in order to evaluate the performance of the tracking controller and the reference trajectory planning policy presented in section 3. The model of the positioning system, regulation feedback and the feedforward tracking compensator are described in section 3. The parameters of x-axis are shown as follows: $B=4\times10^{-4}\,\mathrm{N\cdot m/(rad\cdot s^{-1})}$; $R_a=3.2\,\Omega$; $J_e=1.04\times10^{-4}\,\mathrm{kg\cdot m^2}$; $K_b=5.5\times10^{-2}\,\mathrm{V/(rad\cdot s^{-1})}$; $L_a=2\,\mathrm{mH}$; $K_T=6\times10^{-5}\,\mathrm{N\cdot m\cdot A^{-1}}$; $P_l=20\,\mathrm{mm}$; $A_x=0.005$ V/count; $K_{vx}=0.0117\,\mathrm{V/(rad\cdot s^{-1})}$.

To evaluate the performance of the feedforward-tracking controller solely, a "high"-frequency sinusoidal reference input position is applied to the simulation model. **Figure 5**(a) and (b) represent the reference input position and the system output position obtained, respectively, with and without tracking compensator. These results show the improvement, in terms of tracking performance, acquired by the introduction of the feedforward compensator. As expected, with this compensator the tracking dynamic of the closed-loop system is faster and can follow high-speed reference input without introducing any lag.

Figure 6 illustrates the simulation model for the X-Y positioning system, and individual axial drive system involved in this model as a subsystem. The simulation result of a circle-like ($r=20$ mm) curve that consists of 181 segments and with feedrate 200 mm/s shows in figure 6, figure 6 (a) is the speed curve and acceleration profile after interpolating and figure 6 (b) is the comparison of contour error with compensator and without compensator.

The circle-like curve is interpolated to 525 points to

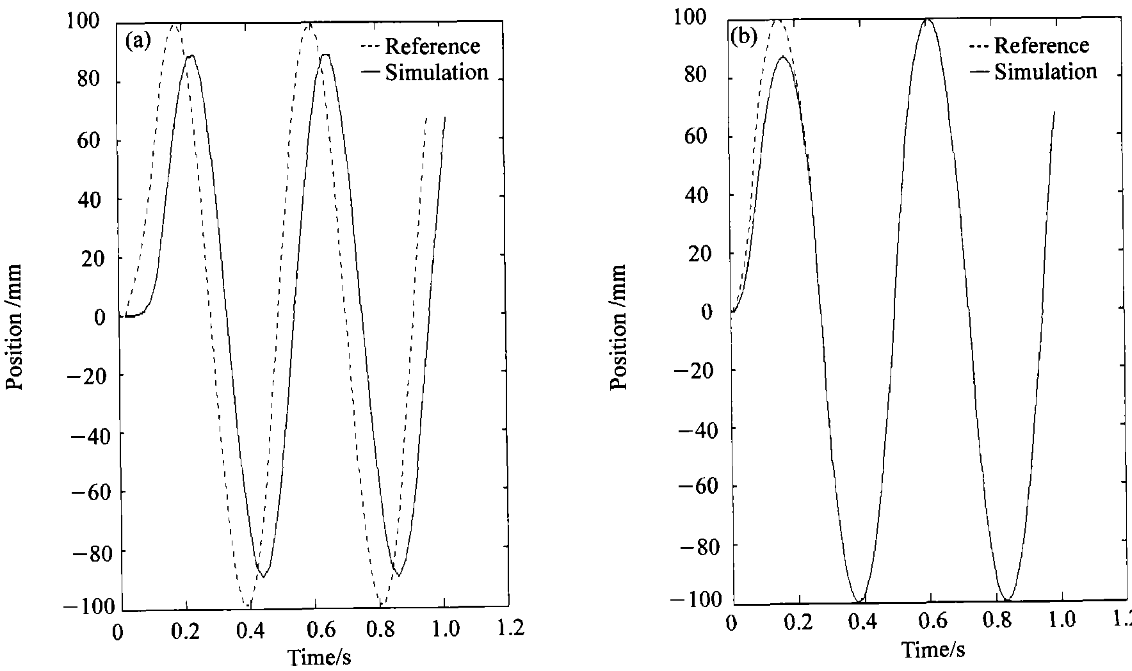

Figure 5 The reference input position and the system output position obtained, (a) With tracking compensator; (b) Without tracking compensator.

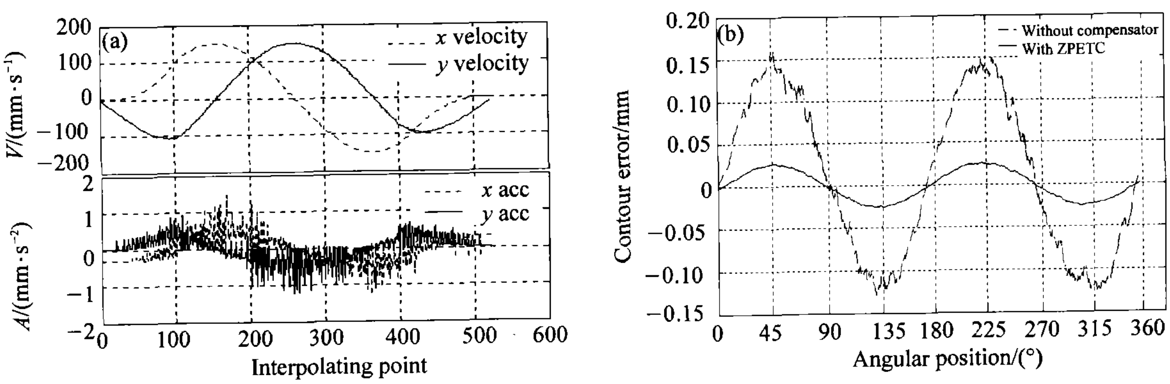

Figure 6 The simulation model for the X-Y positioning system, (a) speed and acceleration profile; (b) contour error.

be tracking. Smooth speed profile is acquired, see figure 6 (a). As shown in figure 6 (b) the *X-Y* positioning system can achieve precise contour control with ZPETC compensator, which eliminates the single axis lag thus reduce the contour error.

5 Conclusion

The control of a LOM machine requires a high precision positioning system. The proposed tracking controller structure and trajectory planning method is useful in LOM application where fast and accurate path following is desired. The resulting motion is smooth with respect to time and has no actuator saturation. The resulting system is linear, so linear techniques still apply to the design of a suitable controller. ZPETC used in this controller eliminates the single axis following error, thus reduces the contour error. According to the simulation of x-axis drive system and the *X-Y* positioning system on a Maltab model, good performance is acquired.

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