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Effect of coherent-light phase on tunneling process

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Abstract: The coherent-light-driven tunneling in double quantum wells has been studied. The electrons are coupled to a system of phonons and subjected to the two beams of coherently optical waves. By adopting a gauge to both the external field and the phonon field, the phonon field operators in the Schrödinger equations are eliminated. In this way, an expression of the tunneling current is conveniently derived considering the relaxation effect. It is shown that under the intense laser field, the tunneling current oscillates rapidly with time at low temperature. The duration of the oscillations is related to the temperature. By adjusting the phase difference of the two light-beams, the oscillation frequency can be modulated.

Key words: tunneling current; quantum wells; gauge transformation

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1 Introduction

Although the photon assisted tunneling processes have been studied extensively [1-6], it is not clear that the phase of the coherent light-beams how to affect the tunneling processes. For the optical waveguides made by the double quantum wells GaAs-AlGaAs-GaAs [7], if the separation of the two waveguides is very closer, there would bring about the electron tunneling between the two waveguides. The changing-tunneling current would affect the device performance. So it is necessary to find out the relation between the phase difference and the tunneling current so as to provide useful information for the device design.

In the tunneling process, the electron-phonon scattering would reduce the electron velocity and affect the tunneling current. The tunneling structure is modeled on the basis of a single-particle picture considering the electron-phonon interaction by proper approximation [8]. Here a new approach is developed to deal with the electron-phonon scattering. This approach is based on the local gauge to both the laser field and the phonon field, the effective Schrödinger equations that do not involve the phonon operators are constructed.

In this paper, the effective Schrödinger equations of describing the tunneling process are derived, and the coherent-light-driven tunneling current in double quantum wells is calculated.

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2 Theoretical analysis

Let two beams of the coherent lights irradiate two wells of the modulation-doped double quantum wells GaAs-AlGaAs-GaAs, respectively, as shown in **figure 1**. Since the external fields interact with the electron gasses, the electron occupations in the two-quantum wells change with time, and generate the tunneling current. In the follows, it is mainly discussed that how the phase difference of the coherent lights affects the tunneling current.

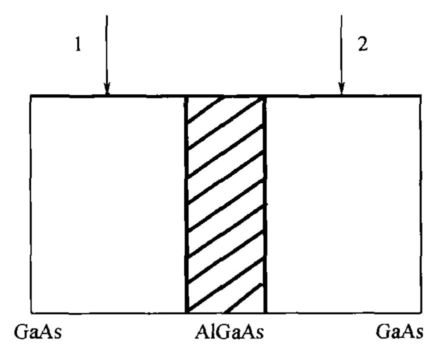

Figure 1 Two coherent light-beams 1 and 2 irradiate the double quantum wells GaAs-AlGaAs-GaAs.

The laser fields are described by the vector potentials of time-dependent:

$$\vec{A}_1 = A_0\vec{e}_x \cos(\omega t) \tag{1}$$

which is the optical wave of irradiating on the left well, and

$$\vec{A}_2 = A_0\vec{e}_x\cos(\omega t+\delta) \tag{2}$$

which is the optical wave of irradiating on the right well. Here $\vec{e}_x$ is the unit vector, δ the phase difference between two optical waves and A_0 the amplitude of the vector potential. The optical waves are assumed to propagate along the z-direction. The spatial dependence of the optical waves is neglected (dipole approximation). According to the general theory of discussing the tunneling effect [9], the Schrödinger equations for the electrons in two wells in the field of the laser light are

$$i\hbar\frac{\partial\Psi_1}{\partial t}=\frac{1}{2m}\left(\vec{P}-\frac{e}{c}\vec{A}_1\right)^2\Psi_1 - i\sum_q v_q\left(a_q e^{i\vec{q}\cdot\vec{r}}-a_q^* e^{-i\vec{q}\cdot\vec{r}}\right)\Psi_1 + B\Psi_2 \tag{3}$$

and

$$i\hbar\frac{\partial\Psi_2}{\partial t}=\frac{1}{2m}\left(\vec{P}-\frac{e}{c}\vec{A}_2\right)^2\Psi_1 - i\sum_q v_q\left(a_q e^{i\vec{q}\cdot\vec{r}}-a_q^* e^{-i\vec{q}\cdot\vec{r}}\right)\Psi_2 + B\Psi_1 \tag{4}$$

where Ψ_1, Ψ_2 are the electron wave functions on the left well and on the right well, respectively; B the coupling matrix element and a_{iq}, a_{iq}^* the annihilation and the creation operators of the phonons in two wells, respectively. The electron-phonon coupling constant v_q is

$$v_q=\left[\frac{2\pi e^2\hbar\omega_L}{V}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right)\right]^{1/2}\frac{1}{q} \tag{5}$$

Let

$$\Psi_1=\Phi_1 e^{i\varphi_1} \tag{6}$$

and

$$\Psi_2=\Phi_2 e^{i\varphi_2} \tag{7}$$

Substitute for the Ψ_1 and Ψ_2 in equations (3) and (4). The following transformations are adopted:

$$-\frac{i\hbar^2}{m}\nabla\varphi_1=\frac{i\hbar e}{2mc}\vec{A}_1 \tag{8}$$

$$-\frac{i\hbar^2}{m}\nabla\varphi_2=\frac{i\hbar e}{2mc}\vec{A}_2 \tag{9}$$

$$-\hbar\frac{\partial\varphi_1}{\partial t}=-\frac{i\hbar^2}{2m}\nabla^2\varphi_1+\frac{\hbar^2}{2m}(\nabla\varphi_1)^2-\frac{e\hbar}{2mc}\vec{A}_1\cdot\nabla\varphi_1+\frac{e^2A_1^2}{2mc^2}-i\sum_q v_q\left(a_q e^{i\vec{q}\cdot\vec{r}}-a_q^* e^{-i\vec{q}\cdot\vec{r}}\right) \tag{10}$$

and

$$-\hbar\frac{\partial\varphi_2}{\partial t}=-\frac{i\hbar^2}{2m}\nabla^2\varphi_2+\frac{\hbar^2}{2m}(\nabla\varphi_2)^2-\frac{e\hbar}{2mc}\vec{A}_2\cdot\nabla\varphi_2+\frac{e^2A_2^2}{2mc^2}-i\sum_q v_q\left(a_q e^{i\vec{q}\cdot\vec{r}}-a_q^* e^{-i\vec{q}\cdot\vec{r}}\right) \tag{11}$$

The phases φ_1, φ_2 satisfying equations (8)-(11) are

$$\varphi_1=-\frac{7e^2A_0^2}{16\hbar mc^2}t+\frac{7e^2A_0^2}{32\hbar\omega mc^2}\sin(2\omega t)+\frac{it}{\hbar}\sum_q v_q\left(a_q e^{i\vec{q}\cdot\vec{r}}-a_q^* e^{-i\vec{q}\cdot\vec{r}}\right) \tag{12}$$

and

$$\varphi_2=-\frac{7e^2A_0^2}{16\hbar mc^2}t+\frac{7e^2A_0^2}{32\hbar\omega mc^2}\sin(2\omega t+2\delta)+\frac{it}{\hbar}\sum_q v_q\left(a_q e^{i\vec{q}\cdot\vec{r}}-a_q^* e^{-i\vec{q}\cdot\vec{r}}\right) \tag{13}$$

Therefore, the Schrödinger equations are rewritten as

$$i\hbar\frac{\partial\Phi_1}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Phi_1+Be^{i(\varphi_2-\varphi_1)}\Phi_2 \tag{14}$$

$$i\hbar\frac{\partial\Phi_2}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Phi_2+Be^{i(\varphi_2-\varphi_1)}\Phi_1 \tag{15}$$

For the response of the electrons to the laser field is faster than that of the phonons, the electrons are seemed to move in the average phonon-potential field. Therefore the statistical average over the phonon operators, $\{a_{iq}\}$ and $\{a_{iq}^*\}$ (i = 1, 2) are used to eliminate phonon operators on both sides of each equation. It is convenient to evaluate the statistical average in the holomorphic representation [10-13]. The average of any operator $B(\{a_{iq}\}\{a_{iq}^*\})$ is expressed as

$$\langle B\rangle_i=\frac{\frac{1}{2\pi i}\int\prod_q \mathrm{d}a_{iq}\mathrm{d}a_{iq}^* B(\{a_{iq}\}\{a_{iq}^*\})e^{-\beta\hbar\omega_L\sum_q a_{iq}^*a_{iq}}}{\frac{1}{2\pi i}\int\prod_q \mathrm{d}a_{iq}\mathrm{d}a_{iq}^* e^{-\beta\hbar\omega_L\sum_q a_{iq}^*a_{iq}}} \tag{16}$$

In this way, the effective Schrödinger equations are obtained by the average to equations (14) and (15):

$$i\hbar\frac{\partial\Phi_1}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Phi_1+B\left\langle e^{i(\varphi_2-\varphi_1)}\right\rangle_{1,2}\Phi_2 \tag{17}$$

$$i\hbar\frac{\partial\Phi_2}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Phi_2+B\left\langle e^{i(\varphi_2-\varphi_1)}\right\rangle_{1,2}\Phi_1 \tag{18}$$

In calculation, the phonons in two wells are considered to be two different subsystems, the corresponding operators $e^{\pm i\varphi_1}$ and $e^{\pm i\varphi_2}$ should be averaged respectively, *i.e.*

$$\langle e^{\pm i\varphi_2 \mp i\varphi_1}\rangle_{1,2} = \langle e^{\pm i\varphi_2}\rangle_2 \langle e^{\mp i\varphi_1}\rangle_1 \tag{19}$$

On evaluating, there have

$$\langle e^{i\varphi_1}\rangle_1 = \exp\left[-i\frac{7e^2A_0^2}{16\hbar mc^2}t + i\frac{7e^2A_0^2}{32\hbar\omega mc^2}\sin(2\omega t) - \sum_q \frac{v_q^2 t^2}{\beta\hbar^3\omega_L}\right] \tag{20}$$

$$\langle e^{i\varphi_2}\rangle_2 = \exp\left[-i\frac{7e^2A_0^2}{16\hbar mc^2}t + i\frac{7e^2A_0^2}{32\hbar\omega mc^2}\cdot [\sin(2\omega t + 2\delta) - \sin(2\delta)] - \sum_q \frac{v_q^2 t^2}{\beta\hbar^3\omega_L}\right] \tag{21}$$

The expansion formula

$$\Phi_1 = \sum_k C_{1k} e^{i\vec{k}\cdot\vec{r}} \tag{22}$$

and

$$\Phi_2 = \sum_k C_{2k} e^{i\vec{k}\cdot\vec{r}} \tag{23}$$

are substituted into equations (17) and (18), there have

$$i\hbar\frac{\partial C_{1k}}{\partial t} = \varepsilon_k C_{1k} + B\langle e^{i(\varphi_2-\varphi_1)}\rangle_{1,2} C_{2k} \tag{24}$$

$$i\hbar\frac{\partial C_{2k}}{\partial t} = \varepsilon_k C_{2k} + B\langle e^{-i(\varphi_2-\varphi_1)}\rangle_{1,2} C_{1k} \tag{25}$$

where $\varepsilon_k = \dfrac{\hbar^2k^2}{2m}$ is the energy of the electron.

Although the phonon operators are eliminated by the statistical average to the phonon system, the effect of the electron-phonon interaction on the electron tunneling exhibits in the coupling term of each equation (the second term on the right side of each equation). The wave functions in $\vec{k}$ space is expressed as

$$C_{1k} = (n_{1k})^{1/2} e^{i\theta_{1k}} \tag{26}$$

and

$$C_{2k} = (n_{2k})^{1/2} e^{i\theta_{2k}} \tag{27}$$

where n_{1k}, n_{2k} are the particle numbers with the wave vector $\vec{k}$ in well 1, 2, respectively. θ_{1k}, θ_{2k} are the electron phases of time-dependent in the two wells. Equations (26) and (27) are substituted into equations (24) and (25), there have

$$i\hbar\frac{\partial n_{1k}}{\partial t} - \hbar n_{1k}\frac{\partial\theta_{1k}}{\partial t} = \varepsilon_k n_{1k} + B\exp\left\{i\frac{7e^2A_0^2}{32\hbar\omega mc^2}\cdot [\sin(2\omega t + 2\delta) - \sin(2\delta) - \sin(2\omega t)] - 2\sum_q \frac{v_q^2t^2}{\beta\hbar^3\omega_L} + i(\theta_{2k} - \theta_{1k})\right\}(n_{1k}n_{2k})^{1/2} \tag{28}$$

and

$$i\hbar\frac{\partial n_{2k}}{\partial t} - \hbar n_{2k}\frac{\partial\theta_{2k}}{\partial t} = \varepsilon_k n_{2k} + B\exp\left\{-i\frac{7e^2A_0^2}{32\hbar\omega mc^2}\cdot [\sin(2\omega t + 2\delta) - \sin(2\delta) - \sin(2\omega t)] - 2\sum_q \frac{v_q^2t^2}{\beta\hbar^3\omega_L} - i(\theta_{2k} - \theta_{1k})\right\}(n_{1k}n_{2k})^{1/2} \tag{29}$$

The any one of the two equations is divided into the real part and the imaginary part. Its real part is

$$\frac{\partial n_{1k}}{\partial t} = -\frac{B}{\hbar} e^{-\frac{2t^2}{\beta\hbar^3\omega_L}\sum_q v_q^2} \sin\{\frac{7e^2A_0^2}{32\hbar\omega mc^2}\cdot [\sin(\omega t)\sin(\delta)\sin(\omega t + \delta)] - (\theta_{2k} - \theta_{1k})\}(n_{1k}n_{2k})^{1/2} \tag{30}$$

Considering that when the phase difference δ is zero, there is no tunneling current, one derives $\theta_{2k} - \theta_{1k} = 0$. For the two wells are identical, there should be $n_{2k} \approx n_{1k}$. Therefore the tunneling current is expressed as

$$I' = -e\sum_k \left\langle \frac{\partial n_{1k}}{\partial t}\right\rangle_{el} = \frac{eB}{\hbar} e^{-\frac{2t^2}{\beta\hbar^3\omega_L}\sum_q v_q^2} \sin\{\frac{7e^2A_0^2}{32\hbar\omega mc^2}\cdot [\sin(\omega t)\sin(\delta)\sin(\omega t + \delta)]\} \tag{31}$$

where $\langle\cdots\rangle_{el}$ is the statistical average to the electron system. Show the summation for $\sum_q v_q^2$, the relative current becomes

$$I = I'\Big/\frac{eB}{\hbar} = e^{-\left(\frac{t}{t_c}\right)^2} \sin\{\frac{7e^2A_0^2}{32\hbar\omega mc^2}\cdot [\sin(\omega t)\sin(\delta)\sin(\omega t + \delta)]\} \tag{32}$$

where

$$t_c \approx 1\Big/\sqrt{\frac{2e^2kT}{\pi\hbar^2}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right)q_{max}} \tag{33}$$

is the damping time of the tunneling current, here q_{max} is the wave vector in the boundary of the first Brillouin zone.

Take the double quantum wells GaAs-AlGaAs-GaAs as an example, the material parameters of the GaAs are chosen as ε_0=12.91, ε_∞=10.91, m=0.067m_0 (m_0 for the static mass of the electron), and the lattice spacing is 0.565 nm. It is found that the damping time t_c of the tunneling current is related to the temperature. The higher the temperature is, the shorter the damping time will be. At 4.2 K, the damping time is t_c=5.72×10^{-13} s. So in order to observe this effect, the temperature must be lower than 4.2 K. On the other hand, in order to achieve the larger tunneling current, the laser field strength E_0 should obey

$$\frac{7e^2E_0^2}{8m\hbar\omega^3} \geq \frac{\pi}{2}, \quad i.e.$$

$$E_0 \geq \left(\frac{4\pi m\hbar\omega^3}{7e^2}\right)^{1/2} \tag{34}$$

From equation (32) it is found that as long as there is one function of zero value in the three functions $\sin\delta$, $\sin(\omega t)$, and $\sin(\omega t+\delta)$, the current is zero. For the double quantum wells GaAs-AlGaAs-GaAs, if the CO_2 laser (wavelength $\lambda=10.6\ \mu m$) is applied to irradiate the two wells, the laser field strength should not be less than the order of 10^5 V/cm.

As shown in **figure 2**, in ultra-short time, the relative tunneling oscillates rapidly with time. The 'oscillation frequency' of the tunneling current induced by the two light-beams with phase difference $\pi/2$ is higher than that with phase difference $\pi/4$. The duration time of the tunneling current changes with temperature, and the 'oscillation frequency' of the tunneling current changes with the phase difference between the two beams of coherent lights.

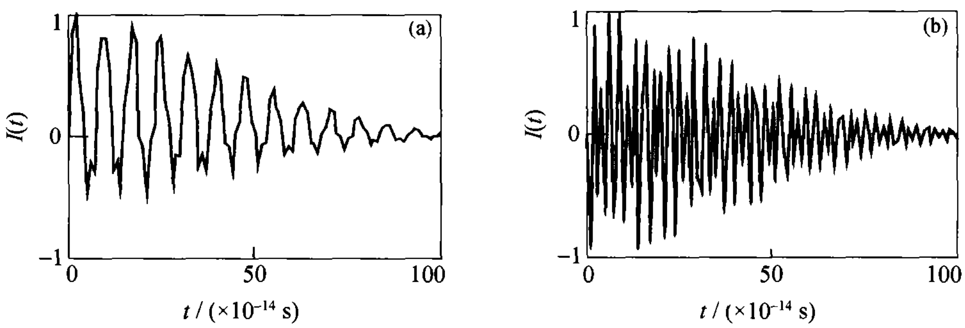

Figure 2 The tunneling current *vs.* time corresponding to the laser field strength E_0=1.0×10^5 V/cm, the phase difference of the two light-beams at the temperature of 4.2 K, (a) $\pi/4$, (b) $\pi/2$.

3 Conclusions

The theoretical investigations have shown that the coherent light-beams can induce a tunneling current with high-speed oscillation. For the double quantum wells GaAs-AlGaAs-GaAs, if the laser field strength is no less than the order of 10^5 V/cm, the tunneling current can be observed at low temperature and in an ultra-short time. The reason of rapidly damping for the tunneling current is due to the electron-phonon interaction.

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