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Dynamic analysis of fault rockburst based on gradient-dependent plasticity and energy criterion

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Abstract: Fault rockburst is treated as a strain localization problem under dynamic loading condition considering strain gradient and strain rate. As a kind of dynamic fracture phenomena, rockburst has characteristics of strain localization, which is considered as a one-dimensional shear problem subjected to normal compressive stress and tangential shear stress. The constitutive relation of rock material is bilinear (elastic and strain softening) and sensitive to shear strain rate. The solutions proposed based on gradient-dependent plasticity show that intense plastic strain is concentrated in fault band and the thickness of the band depends on the characteristic length of rock material. The post-peak stiffness of the fault band was determined according to the constitutive parameters of rock material and shear strain rate. Fault band undergoing strain softening and elastic rock mass outside the band constitute a system and the instability criterion of the system was proposed based on energy theory. The criterion depends on the constitutive relation of rock material, the structural size and the strain rate. The static result regardless of the strain rate is the special case of the present analytical solution. High strain rate can lead to instability of the system.

Key words: strain localization; rockburst; plastic strain gradient; criterion of instability; strain rate; fault band; strain softening

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1 Introduction

Rockburst is one of serious natural hazards in mining engineering [1]. Cook conducted the first theoretical analysis of rockbursts and it was considered as a problem of stability [2]. Since the pioneering work by Cook, many researchers have studied the rockburst based on some kinds of instability theories. For example, Lippman treated rockburst as an instability phenomenon in the sense of limiting static equilibrium of elastic-plastic material [3] and Vardoulakis treated rock burst as a surface instability phenomenon [4].

The problem of rockburst has received considerable attention in recent years and many important results have been achieved in China because the mining depth increases and various geology factors coexist. Some typical theories or techniques have been proposed or adopted in order to understand the mechanism of rockburst so far, such as stability theory [5], the stability of motion [6], the numerical simulation using three dimensional FEM [7,8] and FLAC [9,10], cusp catastrophic theory [11], fracture mechanics method [12], neural network method [13]. Nevertheless, several critical problems on rockburst remain open so far:

(1) Firstly, rockburst belongs to a kind of dynamic phenomena and strain rate has an influence on the instability of rockburst surely. But, some theoretical and numerical analysis were carried out under static loading conditions, which can not reflect completely the essence of the dynamic phenomenon to some extent.

(2) Secondly, observations in field and in laboratory show that the failure of rock are localized in certain location called localized zone [14-17]. Outside the localized zone, the rock is intact and is unloaded in strain softening stage. The important characteristics of localization need to be considered to obtain a full understanding of rockburst mechanism.

(3) Thirdly, the dimension of localized zone can not be determined in the context of classical elastoplastic theory. Where the thickness of the slip line is zero, which is not in agreement with many experimental results [18] that the thickness depends on average grain diameter due to the fact that almost all engineering

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materials have microstructures.

(4) Finally, it has been widely known that violent fracture of rock occurs when an excess of energy becomes available during the post-peak deformation stage. Rockburst occurs when the stiffness of the loading system is softer than the post-peak stiffness of the failing rock specimen [19]. Based on classical elastoplastic theory, the post-peak force-deformation diagrams can not be obtained analytically due to the fact that the dimension of the localized zone is unknown.

In the present paper the important characteristics of localization of rockburst is considered based on gradient-dependent plasticity and the criterion of the dynamic instability of is obtained.

2 Analysis of dynamic fault rockburst

2.1 Basic assumptions and classical elastoplastic theory

The problem of rockburst is shown in **figure 1**. It is a one-dimensional rock block of height $L+w$ that is loaded in shear stress τ in the horizontal direction and is loaded in compressive constant stress σ in vertical direction. The y-axis is chosen parallel to the vertical direction. For simplicity, the lower end of the rock block is fixed (*i.e.* the horizontal and the vertical displacements of the end are zero) and the length of the rock block is long enough. According to many experimental results, it can be supposed that localization is initiated at the peak stress and the shear deformation only occurs in the horizontal direction. The slope of shear stress-shear strain curve up to the peak stress can be considered approximately the same constant G. So, in the elastic stage, the shear elastic modulus G governs the relation between shear stress and elastic shear strain: $\tau = G\gamma$. The static constitutive relation between shear stress and (plastic) shear strain is shown in **figure 2**. During the post peak, the absolute value of the slope of shear stress-shear strain curve can also be considered approximately the constant λ called shear softening modulus and $\lambda > 0$. In general, according to many experimental results, the parameter λ and peak shear stress τ_c are dependent on the confining pressure σ. In the context of one-dimensional conventional plasticity theory for strain softening material loaded or unloaded elastically according to shear elastic modulus G, the flow shear stress τ is an explicit function of shear strength τ_c, λ, G, and the accumulated plastic shear strain γ^p, *i.e.*

$$\begin{cases} \tau = \tau_c - \dfrac{G\lambda}{G+\lambda}\gamma^p = \tau_c - c\gamma^p \\ c = \dfrac{G\lambda}{G+\lambda} \end{cases} \tag{1}$$

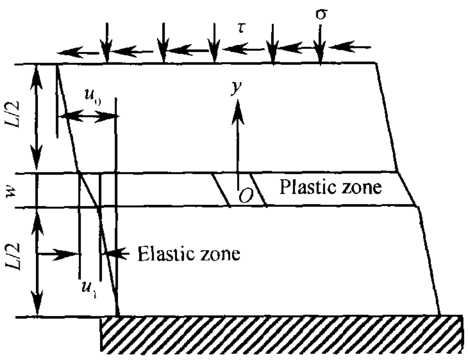

Figure 1 Fault band and elastic zone outside the band.

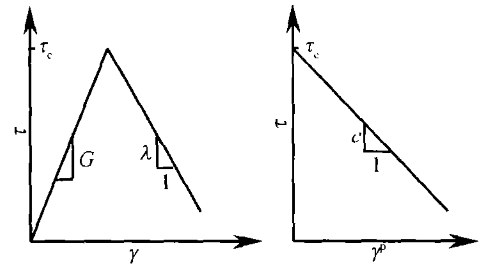

Figure 2 The static shear stress—(plastic) shear strain curve, (a) γ; (b) γ^p.

2.2 Considering the effect of strain rate

To consider the effect of strain rate, involving a function f into the yield function in the context of classical elastoplastic theory yields [20]

$$\tau = \left(\tau_c - \frac{G\lambda}{G+\lambda}\gamma^p\right)\cdot f \tag{2}$$

where $f = 1 + C\ln\dot{\gamma}/\dot{\gamma}_0$, $f \geq 1$; C is a material constant; $\dot{\gamma}$ is average shear strain rate and $\dot{\gamma}_0$ is average shear strain rate under quasi-static conditions. For simplicity, let $\tau_c' = f\tau_c$ and $c' = fc$. The dynamic constitutive relation between shear stress and plastic shear strain is shown in **figure 3**.

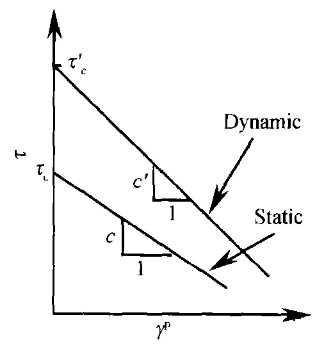

Figure 3 Comparison of dynamic and static constitutive relations.

2.3 Local plastic shear strain in fault band

The following expression can be gotten according to the usual method of introducing the strain gradient into the yield function [21-32], so that the shear strain gradient effect can be investigated in the context of conventional plastic theory.

$$\tau = f\tau_c - fc\left(\gamma^p + l^2\frac{d^2\gamma^p}{dy^2}\right) \tag{3}$$

Suppose that the plastic zone has a width w after localization is initiated. In the boundary of elastic and plastic zones, the boundary condition is:

$$\gamma^p = 0, \text{ at } y = \pm w/2 \tag{4}$$

Considering the plastic shear strain in fault band is an even function and application of the boundary condition above results in:

$$\gamma^p = \frac{\tau_c^{'} - \tau}{c'}\left(1 - \cos\frac{y}{l} \Big/ \cos\frac{w}{2l}\right) \tag{5}$$

The thickness of shear band is determined according to the maximum plastic shear strain:

$$\frac{d\gamma^p}{dw} = 0 \tag{6}$$

Therefore, the thickness of shear band analytically can be obtained

$$w = 2\pi\, l \tag{7}$$

Substitution of equation (7) into equation (5) leads to

$$\gamma^p = \frac{\tau_c^{'} - \tau}{c^{'}}\left(1 + \cos\frac{y}{l}\right) \tag{8}$$

2.4 Application of energy theory

The system shown in figure 1 is composed of two components: one is fault band undergoing strain softening after peak point; the other is linear elastic rock mass outside the fault band. Herein, the fault band is considered to be "specimen". While, the elastic rock mass is seen as "shearing testing machine" with shear elastic modulus G. The horizontal shear displacement between the upper and the lower ends in the shear band (*i.e.* the shear displacement of "specimen") can be obtained

$$u_1 = 2\int_0^{w/2} \gamma dy \tag{9}$$

where $\gamma = \gamma^e + \gamma^p$; $\gamma^e = \tau / G$. Substitution of equation (8) in equation (9) now results in the following expression

$$u_1 = \frac{\tau}{G} w + \frac{\tau_c^{'} - \tau}{c^{'}} w \tag{10}$$

Considering the value of the parameter c, c' and $\tau_c^{'}$, equation (11) can be gotten

$$\tau = \frac{1}{w}\left(\frac{1}{G} - \frac{1}{fc}\right)^{-1} u_1 - \frac{\tau_c}{c}\left(\frac{1}{G} - \frac{1}{fc}\right)^{-1} \tag{11}$$

Supposing the area subjected to shear stress is A, shear force: $F = \tau A$ can be gotten. So the relation between shear force and the horizontal shear displacement between the upper and the lower ends in the fault band can be written as

$$F = \frac{A}{w}\left(\frac{1}{G} - \frac{1}{fc}\right)^{-1} u_1 - \frac{A\tau_c}{c}\left(\frac{1}{G} - \frac{1}{fc}\right)^{-1} \tag{12}$$

For simplicity, let $k = \frac{A}{w}\left(\frac{1}{G} - \frac{1}{fc}\right)^{-1}$. As can be seen that shear force is proportional to u_1 in the strain softening stage. Before the peak stress, strain localization is not initiated, *i.e.* $\gamma^p = 0$, then

$$u_1 = 2\int_0^{w/2} \gamma^e dy = \frac{\tau}{G} w \tag{13}$$

Therefore, in the elastic stage, the relation between shear force and the shear displacement u_1 is

$$F = \frac{GAu_1}{w} \tag{14}$$

A bilinear F—u_1 curve is depicted in **figure 4**. After strain localization is initiated, the slope F—u_1 of curve is k. The absolute value $|k|$ is the post-peak stiffness of the failing rock "specimen". It can be found that the slope of the bilinear curve depends on constitutive parameters of rock material and shear strain rate. Outside of the localized zone, the rock mass is intact or elastic. The elastic shear strain γ^e after the peak stress in elastic zone is

$$\gamma^e = \frac{u_0 - u_1}{L} \tag{15}$$

where u_0 is the relative shear displacement between the upper boundary of the upper elastic rock and the fixed lower boundary of the lower elastic rock (*i.e.* the shear displacement of "shearing testing machine") shown in figure 1. According to shear Hooke's law, the shear stress in the elastic zone is

$$\tau = G\gamma \tag{16}$$

Thus, the shear force F can be obtained as follows

$$F = A\tau = \frac{AG(u_0 - u_1)}{L} = K(u_0 - u_1) \tag{17}$$

where K is a parameter called the shear stiffness of "shearing testing machine". In the system composed of the fault band and rock mass outside the band, the work done by external force is

$$W = \int_0^{u_1} F du_1 = \int_0^{u} K(u_0 - u_1) du_1 = Ku_0u_1 - 0.5Ku_1^2 \tag{18}$$

In the shear band, the sum of elastic strain energy U_E and dissipated strain energy U_S are equal to the area below the curve $F(u_1)$—u_1 shown in figure 4,

$$U_E + U_S = \int_0^{u_1} F(u_1)\mathrm{d}u_1 \tag{19}$$

Therefore, the total potential energy of the system can be gotten

$$\Pi = U_E + U_S - W = -Ku_0u_1 + \frac{1}{2}Ku_1^2 + \int_0^{u_1} F(u_1)\mathrm{d}u_1 \tag{20}$$

Based on energy theory, the equilibrium condition is

$$\begin{cases} \dfrac{\mathrm{d}\Pi}{\mathrm{d}u_1} = 0 \\ F = F(u_1) \end{cases} \tag{21}$$

The unstable condition of the system is

$$\begin{cases} \dfrac{\mathrm{d}^2\Pi}{\mathrm{d}u_1^2} \le 0 \\ K + F'(u_1) \le 0 \end{cases} \tag{22}$$

According to equation (12) or figure 4, the following equation can be proposed

$$F'(u_1) = \frac{A}{w}\left(\frac{1}{G} - \frac{1}{fc}\right)^{-1} = k \tag{23}$$

According to equation (17), equation (22) and equation (23), the instability criterion of the system can be obtained.

$$\frac{G}{L} + \frac{1}{w}\left(\frac{1}{G} - \frac{1}{fc}\right)^{-1} < 0 \tag{24}$$

If the effect of strain rate is neglected, *i.e.* let $f = 1$, then the equation (24) is simplified as follows

$$\frac{G}{L} < \frac{\lambda}{w} \tag{25}$$

The equation (25) is consistent with static analytical result for fault rockburst in reference [33]. Herein, only the effect of strain rate on the instability of the system will be analyzed. Generally, the slope of post-peak force-deformation curves is negative in strain softening stage. According to equation (18), the absolute value $|k|$ can be gotten

$$|k| = \frac{A}{w}\left(\frac{1}{fc} - \frac{1}{G}\right)^{-1} > 0 \tag{26}$$

As can be seen from equation (26) that the larger value of function f (or the larger shear strain rate), the larger the value of the post-peak stiffness of the fault band $|k|$ is. Increasing strain rate leads to instability of system.

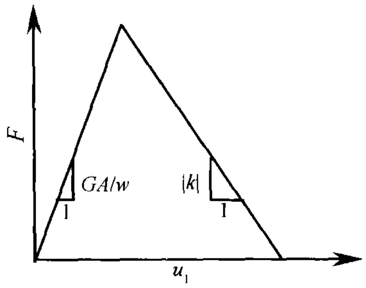

Figure 4 The shear slip constitutive relation between shear force and shear displacement for fault band.

3 Conclusion

(1) Dynamic rockburst has characteristics of strain localization. Uniform failure mode never be observed in experimental and field tests. Intense shear strains are localized in narrow fault band whose thickness depends on the characteristic length of rock material.

(2) The post-peak stiffness of the fault band is dependent on the constitutive parameters of rock material and shear strain rate. Based on the energy criterion, the proposed instability criterion of the system composed of fault band and elastic rock outside the band depends on the constitutive relation of rock materials, the structural size and the shear strain rate. High strain rate can lead to instability of the system. Simplifying the present dynamic instability criterion leads to the early static analytical solution.

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