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A multi-objective decision-making method for the treatment scheme of landslide hazard

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Abstract: The treatment engineering of landslide hazard is a complicated system engineering. The selecting treatment scheme is influenced by many factors such as technology, economics, environment, and risk. The decision-making of treatment schemes of landslide hazard is a problem of comprehensive judgment with multi-hierarchy and multi-objective. The traditional analysis hierarchy process needs identity test. The traditional analysis hierarchy process is improved by means of optimal transfer matrix here. An improved hierarchy decision-making model for the treatment of landslide hazard is set up. The judgment matrix obtained by the method can naturally meet the requirement of identity, so the identity test is not necessary. At last, the method is applied to the treatment decision-making of the dangerous rock mass at the Slate Mountain, and its application is discussed in detail.

Key words: landslide hazard; treatment scheme; improved hierarchy decision-making model; optimal transfer matrix

1 Introduction

The decision-making for the treatment scheme of landslide hazard is a problem of comprehensive judgment with multi-objective and multi-hierarchy [1, 2]. The judgment made through qualitative and quantitative methods to the treatment scheme of landslide hazard is a key to the decision-making of the treatment scheme of landslide hazard. The analysis hierarchy process proposed by T.L. Satty is a good method, which combines qualitative analysis with quantitative analysis [3]. The method has a good effect on solving the multi-objective and multi-hierarchy optimization problem of the treatment scheme of landslide hazard. It is highly logical, flexible and concise. Its basic principle is to compare the relative importance of every two evaluating factors with each other and then ordered them [3-5]. Because of artificial one-side judgment, the comparative results are not to be objective identity, then the identity test is often needed [6,7]. If it does not stand the test, the traditional method is that the judgment matrix would be adjusted by approximate estimation. Though it is effective, it is sometimes blind and subjective. It needs to adjust many times in order to stand the identity test. The traditional analytic hierarchy process is improved by the optimal transfer matrix [8], which meets the requirement of identity. In this article, the treatment decision-making of a dangerous rock mass at the Slate Mountain is taken as an example [9-12], and the application of the improved analytic hierarchy process method in the multi-objective and multi-hierarchy decision-making for the treatment scheme of landslide hazard is discussed in detail.

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2 Improved analysis hierarchy principle and model

Suppose real number matrixes $\boldsymbol{A}=[a_{ij}]$, $\boldsymbol{B}=[b_{ij}]$, $\boldsymbol{C}=[c_{ij}]\in\mathbf{R}^{n\times n}$.

Definition 1 If $a_{ij}=1/a_{ji}$, $\boldsymbol{A}$ is called mutual antimatrix; if $b_{ij}=-b_{ji}$, $\boldsymbol{B}$ is called antisymmetry matrix.

Definition 2 If $\boldsymbol{A}$ is a mutual antimatrix, and $b_{ij}=-b_{ji}$, $\boldsymbol{A}$ is identical; if $\boldsymbol{B}$ is an antisymmetry matrix, and $b_{ij}=b_{ik}+b_{kj}$, $\boldsymbol{B}$ is transferred.

Definition 3 If there exists a transfer matrix $\boldsymbol{C}$, and it makes $\sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij}-b_{ij})^2$ minimum, $\boldsymbol{C}$ is called $\boldsymbol{B}$'s optimal transfer matrix.

Theorem 1 If $\boldsymbol{B}$ is an antisymmetry matrix, $\boldsymbol{B}$'s optimal transfer matrix $\boldsymbol{C}$ must meet the requirement:

$$c_{ij}=\frac{1}{n}\sum_{k=1}^{n}(b_{ik}-b_{jk}).$$

Theorem 2 If $\boldsymbol{A}$ is a mutual antimatrix, and $\boldsymbol{B}=\lg\boldsymbol{A}$, and $\boldsymbol{C}$ is $\boldsymbol{B}$'s optimal transfer matrix, the matrix $\boldsymbol{A}^*=10^{c}$ is an $\boldsymbol{A}$'s matching optimal transfer matrix, and it is identical.

In the traditional analytic hierarchy process, the judgment matrix $A=[a_{ij}]_{n\times n}$ produced by scale method has the following characters: $a_{ij}>0$; $a_{ij}=1/a_{ji}$; $a_{ii}=1$. Obviously, A is a mutual antimatrix, forming a matrix $A^*=10^{c_{ij}}$, where $c_{ij}=\frac{1}{n}\sum_{k=1}^{n}(b_{ik}-b_{jk})$.

From theorem 2, it is known that matrix A^* is A's matching optimal transfer matrix, and it is identical. So the weighing value (the character value of A^*) can be obtained directly from A^* without an identity test.

The character value of A^* is calculated by root-square method. Firstly, the matrix itself has relative errors, so its character vector does not need much high precision. Then, the root-square method is actually to assess weighing values by the super-geometric average method and it can make the character value of the matching optimal transfer matrix A^* closer to the optimal transfer matrix's character value.

3 A multi-objective decision-making method for the treatment scheme of landslide hazard

3.1 Define the weighing matrix $W^{(p)}$ of evaluation indexes

If there are m evaluation indexes in the decision problem of the treatment scheme of landslide hazard, and there are r experts to give their evaluation, the evaluation index set of the slope treatment question is $X=\{x_1,x_2,\cdots,x_m\}$, and the expert set is $S=\{s_1,s_2,\cdots,s_r\}$ (m, $r\geq1$). Adopting the two-two comparative method, the expert who is included in set S assesses every weighing index, X_i and x_j are compared with each other by any expert S_k in expert set S, and can form a comparative matrix:

$\boldsymbol{D}^{(k)}=[d_{ij}^{(k)}]_{m\times m}$,

where

$$d_{ij}^{(k)}=\begin{cases}2 & x_i \text{ is more important than } x_j\\ 1 & x_i \text{ is as important than } x_j\\ 0 & x_j \text{ is more important than } x_i\end{cases}$$

and there $d_{ii}^{(k)}=1$, that is the objective X_i comparing with itself, having the same importance.

According to the order of its importance degree, the judgment matrix can be formed as:

$A^{(k)}=\left[a_{ij}^{(k)}\right]_{m\times m}$,

where

$$a_{ij}^{(k)}=\begin{cases}r_i^{(k)}-r_j^{(k)} & r_i>r_j\\ 1 & r_i=r_j\\ [r_j^{(k)}-r_i^{(k)}]^{-1} & r_i<r_j\end{cases}$$

$r_i^{(k)}=\sum_{l=1}^{m}d_{il}^{(k)}$ $\quad(i=1,2,\cdots,m;\ k=1,2,\cdots,r)$.

Obviously, $A^{(k)}\sigma_{ij}$ is a mutual antimatrix, $\boldsymbol{B}^{(k)}=\lg A^{(k)}=[\lg a_{ij}^{(k)}]_{m\times m}$ is an antisymmetry matrix. If σ_{ij} is the total standard error of expert's evaluation, there is

$$\sigma_{ij}=\sqrt{\tfrac{1}{r-1}\sum_{l=1}^{r}[b_{ij}^{(l)}-\tfrac{1}{r}\sum_{t=1}^{r}b_{ij}^{(t)}]^2}\ .$$

From the calculation results of σ_{ij}, there are two following situations:

(1) if $\sigma_{ij}<1$, it shows that the opinion of the experts is comparatively unified, and the arithmetic mean of the expert judgment value is regarded as the judgment result of the group set. We can gain

$\boldsymbol{B}=[b_{ij}]_{m\times m}$,

where $b_{ij}=\frac{1}{r}\sum_{l=1}^{r}b_{ij}^{(l)}$ $\quad(i,j=1,2,\cdots m)$.

It does not necessarily have the identity. Applying the improved analysis hierarchy process, the matrix is formed as:

$A^*=[a^*]_{m\times m}$,

where $a^*=10^{c_{ij}}=10^{\frac{1}{m}\sum_{l=1}^{m}(b_{il}-b_{jl})}$.

From theorem 2, it is known that A^* is A's optimal transfer matrix and it is identical. The correspondent vector of A^*'s maximum character value gained by using the root-square method is the index's weighing $\overline{w}=(\overline{w},\overline{w}_2,\cdots,\overline{w}_m)$, and it is normalized, then it changes to be the standard weighing $w=(w_1,w_2,\cdots w_m)$, where $w_i=\overline{w}_i\Big/\sum_{k=1}^{m}\overline{w}_k$, it is the standard weighing of every index.

(2) if $\sigma_{ij}>1$ $(i,j=1,2,\cdots,m)$, it shows that the opinion of the expert group is very different, the arithmetic mean of every expert's judgment value can not be regarded as the judgment result of the group set. But the optimal transfer matrix can be adopted in calculation, that is, there exists an optimal transfer matrix $\boldsymbol{B}=[b_{ij}]_{m\times m}$, make $J=\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{l=1}^{r}[b_{ij}-b_{ij}^{(l)}]^2$ minimum, where

$$b_{ij}=\tfrac{1}{m}\sum_{t=1}^{m}\sum_{l=1}^{r}[b_{it}^{(l)}-b_{jt}^{(l)}]^2 \quad (i,j=1,2,\cdots,m).$$

Let $C=[a^{b_{ij}}]_{m\times m}=[c_{ij}]_{m\times m}$, where a is the objectively important ratio of close grade assessment, and there a=1.1-1.3. The matrix $\boldsymbol{C}$ is the comparative judgment matrix of the group set gained, and it is identical. The index's weighing can be obtained according to the situation (1) in section 3.2.

Calculate the weighing value of each evaluation index corresponding to the upper layer index by the above method, then the weighing of the last layer indexes corresponding to the first layer indexes can be acquired by the hierarchically total order, the standard weighing $w=(w_1,w_2,\cdots,w_m)$ of the evaluation indexes can be gained after normalization.

If some slope has p probable treatment schemes, forming the set $F=\{F_1, F_2, \cdots, F_p\}$. Using the above method can get the standard weighing of the h scheme's evaluating index:

$$w^{(h)}=(w_1^{(h)},w_2^{(h)},\cdots,w_m^{(h)}).$$

The index weighing matrix made up of p schemes is:

$$W^{(p)}=[w_i^{(h)}]_{p\times m}, (i=1,2,\cdots, m; h=1,2,\cdots, p).$$

3.2 Define character matrix $T^{(p)}$ of evaluation index

As to P treatment schemes having m evaluation indexes, the character value of individual evaluation index is $X^i=[x_{ih}^i]_{m\times p}$. The treatment scheme's evaluation involves qualitative and quantitative indexes, which can be settled by the following method.

(1) The standardization of quantitative indexes.

For the index being better if its value is bigger, let

$$\gamma_{ih}^i=x_{ih}^i\Big/\max_{1\le h\le p}\{x_{ih}^i\}.$$

For the index being better if its value is smaller, let

$$\gamma_{ih}^i=(\max_{1\le h\le p} x_{ih}^i-x_{ih}^i)\Big/(\max_{1\le h\le p} x_{ih}^i-\min x_{ih}^i).$$

(2) The standardization of qualitative indexes.

The language variable mark method is adopted in the standardization of the qualitative indexes. The character value of some index in the treatment scheme can be evaluated according to the different grades: excellent, good, ordinary, bad and very bad. If the remark set of some indexes in the treatment scheme may show $R_i=(r_1, r_2, \cdots, r_q)$ (having q grades) and the value is given according to linear equal difference, the quantitative grade matrix of a comment set is $C_i=[1,\frac{n-2}{n-1},\frac{n-3}{n-1},\cdots,0]$. Therefore, when an individual expert gives the remark of the qualitative index x_{ih}^i is $r_i\in R_i$, the normalized character value of this index is

$$\gamma_{ih}^i=\frac{n-i}{n-1}, (n=1, 2,\cdots, q; i=1, 2,\cdots, m).$$

(3) The character matrix of assessment indexes.

The character matrix $\boldsymbol{T}^{(p)}$ of the evaluation index after index normalization is $\boldsymbol{T}^{(p)}=[\gamma_{ih}^i]_{m\times p}$ $(i=1, 2,\cdots, m; h=1, 2,\cdots, p)$.

3.3 Evaluation matrix Z of the treatment scheme

The decision-making evaluation matrix $\boldsymbol{Z}$ of p treatment schemes having m evaluation indexes is

$$Z=W^{(p)}\times T^{(p)}=[z_h]_{1\times p}.$$

The slope optimal treatment scheme can be defined conveniently according to the evaluation matrix **Z**.

4 Engineering example

The side facing the sky of the dangerous rock mass at the Slate Mountain is steep slope or cliff. There are landslides, factories, schools and inhabitant below the scar. The environment is very complex. From the 1950s, geologic disasters of different scales have happened, threatening seriously the factories and residents, so the state and the local government decide to put the dangerous rock mass under control. On the base of geologic exploration and stability analysis, five possible treatment schemes are put forward as: blasting unloading method, drainage works method, flexible reinforcement method, rock-control drain method and anti-sliding concrete key method. In order to find out the better treatment scheme suitable for the critical rock mass, these following indexes are chosen to be the standards of weighing the treatment schemes after analyzing and studying: total investment (x_1), annual maintenance cost (x_2), control effect (x_3), construction difficulty degree (x_4), construction period (x_5), risk (x_6), engineering validity term (x_7), environment affection (x_8). Taking as an example, the analysis of assessment indexes of the blasting unloading method is described as follows: the expert number r=10, according to the evaluation of 10 experts on the evaluation indexes of the blasting unloading method by three-scale method, $\boldsymbol{D}^{(1)}$ and $r_i^{(1)}$ can be obtained, and the judgment matrix $\boldsymbol{A}^{(1)}$ is also derived according to $r_i^{(1)}$. The anti-symmetry matrix $\boldsymbol{B}^{(1)}$ can be calculated through $\boldsymbol{B}^{(1)}$= lg$\boldsymbol{A}^{(1)}$. The total standard deviation is $\sigma_{ij}<1$ after calculation. So the opinion of the evaluation expert set is very concentrated, and the average matrix $\boldsymbol{B}$ of the anti-symmetry matrix $\boldsymbol{B}^{(k)}$ is also calculated. Trough matrix $\boldsymbol{B}$, the marching optimal transfer matrix $\boldsymbol{A}^*$ can be gained. $\boldsymbol{A}^*$'s character vector by the root-square method is: $\overline{w}^{(1)}$=(1.149, 0.085, 1.473, 0.536, 0.532, 1.746 1.329, 0.029). After normalization, the evaluation index weighing of the blasting unload method (F_1)

is $W^{(1)}$=(0.17, 0.01, 0.21, 0.08, 0.08, 0.25, 0.19, 0.01). The other schemes' evaluation index weighing can be gained respectively by the same method as following:

$W^{(2)}$=(0.19, 0.25, 0.11, 0.04, 0.14, 0.11, 0.04, 0.12);

$W^{(3)}$=(0.16, 0.20, 0.08, 0.19, 0.14, 0.12, 0.09, 0.02);

$W^{(4)}$=(0.20, 0.21, 0.06, 0.08, 0.08, 0.19, 0.11, 0.07);

$W^{(5)}$=(0.23, 0.21, 0.10, 0.07, 0.11, 0.13, 0.04, 0.11).

The evaluating index weighing matrix of the treatment scheme of the dangerous rock mass is:

$$\boldsymbol{W}^{(5)} = \begin{bmatrix} 0.17 & 0.01 & 0.21 & 0.08 & 0.08 & 0.25 & 0.19 & 0.01 \\ 0.19 & 0.25 & 0.11 & 0.04 & 0.14 & 0.11 & 0.04 & 0.12 \\ 0.16 & 0.20 & 0.08 & 0.19 & 0.14 & 0.12 & 0.09 & 0.02 \\ 0.20 & 0.21 & 0.06 & 0.08 & 0.08 & 0.19 & 0.11 & 0.07 \\ 0.23 & 0.21 & 0.10 & 0.07 & 0.11 & 0.13 & 0.04 & 0.11 \end{bmatrix}.$$

After expert's quantification and standardization, the evaluating index character matrix of treatment schemes of the dangerous rock mass is

$$\boldsymbol{T}^{(5)} = \begin{bmatrix} 0.50 & 0.50 & 0.25 & 0.75 & 0.25 \\ 0.25 & 0.00 & 0.75 & 0.00 & 0.75 \\ 0.75 & 0.75 & 0.75 & 0.50 & 0.50 \\ 0.25 & 0.25 & 0.50 & 0.25 & 0.50 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 0.50 & 0.50 & 0.75 & 0.50 & 0.75 \\ 0.75 & 0.75 & 1.00 & 0.75 & 1.00 \\ 0.75 & 0.75 & 0.25 & 0.75 & 0.25 \end{bmatrix}.$$

So the decision evaluation matrix of treatment schemes of the dangerous rock mass is

$\boldsymbol{Z} = \boldsymbol{W}^{(P)} \times \boldsymbol{T}^{(P)}$=(0.65, 0.52, 0.62, 0.49, 0.58).

According to the evaluation matrix **Z**, there are two relatively better schemes in the treatment of the dangerous rock mass: the blasting unloading method and the flexible reinforcement method. But according to the geological documents and the in-situ situation, it is known that the loose rock mass's thickness of the dangerous rock mass at the Slate Mountain is relatively large, the rock bed is thin and the dip angle is small. The height of the front cliff of the dangerous rock mass is about 20-50 m. As to the situation, the flexible reinforcement skill is widely used in the reinforcement of the rock and soil mass. But because the thickness of the loose rock mass caused by the slide of the dangerous rock mass is larger than the current validity reinforcement depth, and the reinforcement effect does not come up to the expectation. Because of the thin rock bed and the small dip angle, the rock-bolt runs nearly parallel to the rock bed, which make it difficult to develop the reinforcement effect. The more important thing is that the construction difficulty is huge under the dangerous situation where there is great potential for collapsing and sliding, and the precipitous cliff and underneath steep slopes are unfavorable factors for construction. Therefore, the blasting unloading method should be mainly adopted in the treatment of the dangerous rock mass at the Slate Mountain, at the same time, the dome plastics and anti-permeation methods are also adopted properly. In this way, the optimal treatment scheme of the dangerous rock mass at the Slate Mountain can be set up.

5 Conclusions

Because of the complexity of landslide hazard systems and the diversity of people's knowledge in the concerned field, there exist various ways for the evaluation of landslide hazard. The traditional analytic hierarchy process is developed by means of the optimal transfer matrix. The improved analysis hierarchy method for the treatment decision-making of landslide hazard is set up. The method goes through a series of changes, building a judgment matrix which can meet the demand of the identity, and it seems to set up a regular, adjusting landslide system's identity of people's recognition. Compared with the traditional analytic hierarchy process, it has the advantages such as quickness, effectiveness, simplicity, *etc*. The method is also fit for the decision-making for the treatment schemes of other landslide hazard projects.

References

[1] Y.Y. Xia and R.G. Zhu, Multiperson multilayer fuzzy comprehensive decision making for control plans of unstable slopes [J], *Natural Disasters* (in Chinese), 8(1998), No.1, p.88.

[2] Y.Y. Xia and R.G. Zhu, A system for multiperson multilayer fuzzy comprehensive evaluation of rock slope stability [J], *Eng. Geol.* (in Chinese), 7(1999), No.1, p.46.

[3] S.B. Xu, *The Analytic Hierarchy Principle* [M], Tianjing University Press, Tianjing, 1988.

[4] F. Zahedi, The analytic hierarchy process—a survey of the method and its application [J], *Interaces*, 16(1986), No.4, p.18.

[5] P.T. Hark, Special issue on AHP [J], *Socio-Econ. Plan. Sci.*, 20(1986), No.6, p.57.

[6] Y.Y. Xia, Choosing optimum control plans for unstable slopes by using the analytic hierarchy process (AHP) [J], *Landslide Hazard and Control* (in Chinese), 8(1997), No.3, p.1.

[7] Q.M. Xie and Y.Y. Xia, The hierarchy model of judgement system of treatment schemes of slopes and its application [J], *Rock Mech. Eng.* (in Chinese), 20(2001), Supp.1, p.1065.

[8] Y. Pan and W.M. Wu, Optimal transfer matrix of antima-

trix [J], *Syst. Eng. Theor. Pract.* (in Chinese), 8(1998), No.2, p.44.

[9] M.Y. Liu and R.G. Zhu, Case-based reasoning for slope stability evaluation and its application [J], *J. Wuhan Univ. Technol.* (Materials Science Edition), 16(2001), No.4, p.62.

[10] Y.Y. Xia, Y.M. Xie, and R.G. Zhu, An engineering geology evaluation method based on an artificial neural network and its application [J], *Int. J. Eng. Geol.*, 47(1997), p.149.

[11] Y.Y. Xia and R.G. Zhu, Decision-making analysis of the control project of the landslide hazard of the Banyan Mountain in Huangshi and control [J], *Rock Mech. Eng.* (in Chinese), 19(2000), No.4, p.498.

[12] Q.M. Xie and R.G. Zhu, Grey classification for evaluating the stability of dangerous rock-block masses [J], *J. Wuhan Univ. Technol.* (Materials Science Edition), 15(2000), No.1, p.73.