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Dynamic modeling and analysis of the closed-circuit grinding-classification process

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Abstract: Mathematical models of the grinding process are the basis of analysis, simulation and control. Most existent models including theoretical models and identification models are, however, inconvenient for direct analysis. In addition, many models pay much attention to the local details in the closed-circuit grinding process while overlooking the systematic behavior of the process as a whole. From the systematic perspective, the dynamic behavior of the whole closed-circuit grinding-classification process is considered and a first-order transfer function model describing the dynamic relation between the raw material and the product is established. The model proves that the time constant of the closed-circuit process is lager than that of the open-circuit process and reveals how physical parameters affect the process dynamic behavior. These are very helpful to understand, design and control the closed-circuit grinding-classification process.

Key words: closed-circuit grinding-classification process; open-circuit grinding process; dynamic model; transfer function; time constant; pole analysis; disturbance rejection

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1 Introduction

Mathematical models of the grinding process are the basis of its analysis, simulation and control. Due to the complexity, characters of the grinding process are not yet very well known. It is arduous and not necessary to build a model to describe every aspect of the process. Based on different assumptions many mathematical models of the grinding process have been presented. To model the mill there are the matrix model, the first-order kinetic model, and the perfect mixing model [1-2]. On the basis of these basic models, some new models for different uses were proposed, such as the autogenous grinding model [3] and the cement grinding model [4]. To model the hydrocyclone there are the equilibrium orbit model, the residence time model and the turbulent two-phase flow model [5-6]. These theoretical models are mainly used for computer simulation and are too complex to analyze. Meanwhile some researchers developed models based on identification methods using the input-output data. A transfer function model was frequently derived from the step response of the real process or the computer simulation. Such model has been used in many control methods, such as model predictive control [7], multivariable control [8], and supervisory control [9]. Identification models are comparatively simple and are easier for the controller design. However, identification models are black-box models describing no physical principles inside the process, and the parameters of the models have no direct functional relation with the physical parameters of the process, so they are inconvenient to analyze.

What is the difference between the closed-circuit process and the open-circuit process? How do various physical parameters affect the dynamic behavior of the closed-circuit process? Some answers to such questions have been obtained from experience, but the answers fail to be proved in theory owing to the lack of a convenient model for analysis. The theoretical analysis is indispensable for the further understanding of the laws inside the process and beneficial to design and control as well. Since the dynamic behavior of the closed-circuit process depends on not only the mill but also the hydrocyclone, it is necessary to consider the closed-circuit process as a whole. Many models, however, pay so much attention to the details of the mill or the hydrocyclone that they present little analysis from the systematic perspective.

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The purpose of this paper is to model and analyze the whole closed-circuit grinding-classification process from the systematic perspective. A first-order transfer function is derived. The input of the model is the mass flow of the rough ores fed into the mill and the output is the mass flow of the fine particles in the hydrocyclone overflow. The model describes the dynamic relation between the raw material and the product. The pole analysis of the transfer function proves that the time constant of the closed-circuit process is lager than that of the open-circuit process and reveals how various physical parameters affect the process dynamic behavior.

2 Dynamic model

Figure 1 is a typical closed-circuit grinding-classification process composed of a ball mill and a hydrocyclone. The function of the ball mill is to minimize the particle size and the function of the hydrocyclone is to separate the fine particles from the coarse particles. The fine particles in the hydrocyclone overflow are the products of the grinding process while the coarse particles return to the mill to be ground again. For analysis, the whole closed-circuit grinding-classification process is represented by the block diagram shown in **figure 2.**

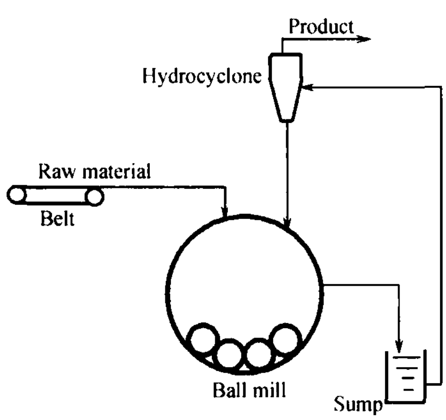

Figure 1 A typical closed-circuit grinding-classification process.

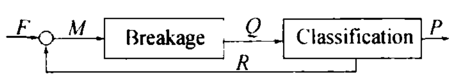

Figure 2 Block diagram of the closed-circuit grinding-classification process.

Since the sump is often controlled by a liquid level controller, the change of its volume is marginal and the time constant is small, so the sump is omitted in figure 2. F is the mass flow of particles in the raw material, M the mass flow of particles into the mill, Q the mass flow of particles out of the mill, P the mass flow of particles in the hydrocyclone overflow, and R the mass flow of particles in the hydrocyclone downflow. Let F_i, A_i, Q_i, P_i, R_i denote the mass flow of particles in size class i of F, M, Q, P, R, respectively. The perfect mixing model [1-2] is convenient to describe the dynamic behavior of the mill because it is a lumped parameter model.

$$\frac{\mathrm{d}s_i(t)}{\mathrm{d}t}=M_i-Q_i+\sum_{j=1}^{i-1}a_{ij}r_js_i-(r_is_i-a_{ii}r_is_i) \tag{1}$$

where $s_i(t)$ is the mass of particles in size class i at time t within the mill, η the rate at which particles in size class i break, a_{ij} the breakage distribution which describes the fraction of particles breaking into size class i due to the breakage of size class j.

In order to simplify the analysis, only two size classes are considered: one is the size class of coarse particles larger than a standard size and the other is the size class of fine particles smaller than the standard size. The former is denoted by the size class A and the latter is denoted by the size class B. The method to describe particles using two size classes is common in practice because of simplicity. Let z_{AF}, z_{AM}, z_{AQ}, z_{AP}, z_{AR} denote the percentage of coarse particles and F_A, M_A, Q_A, P_A, R_A denote the mass flow of the coarse particles. In common cases, the particles in raw material are all coarse, so $z_{AF}=1$. From equation (1) the dynamic function of the breakage of coarse particles inside the mill is

$$\frac{\mathrm{d}s_A}{\mathrm{d}t}=M_A-Q_A-K_Ts_A \tag{2}$$

where $K_T=(1-a_{AA})r_A$, denoting the transformation rate for the coarse particles to break into the fine particles.

In the analysis below, two assumptions are of critical importance [1, 3].

Assumption 1 (Perfect mixing of the mill): Every place inside the mill has the equal concentration and the equal particle distribution.

Assumption 2 (Constant holdup of the mill): The outflow of the mill is equal to the inflow.

Based on the two assumptions,

$$S_A=Vz_{AQ} \tag{3}$$

where V is the volume of particles inside the mill. Substitute equation (3) into (2) to obtain

$$V\frac{\mathrm{d}z_{AQ}}{\mathrm{d}t}=Mz_{AM}-Qz_{AQ}-K_TVz_{AQ} \tag{4}$$

Since $Q_A=Qz_{AQ}$, then

$$\frac{\mathrm{d}z_{AQ}}{\mathrm{d}t}=\frac{1}{Q}\frac{\mathrm{d}Q_A}{\mathrm{d}t}-\frac{Q_A}{Q^2}\frac{\mathrm{d}Q}{\mathrm{d}t} \tag{5}$$

Substitute equation (5) into (4) and obtain

$$\frac{dQ_A}{dt}-\frac{Q_A}{Q}\frac{dQ}{dt}=\frac{Q^2}{V}z_{AM}-\frac{QQ_A}{V}-K_TQ_A \quad (6)$$

Because the time constant of the hydrocyclone is much smaller than that of the mill [1], the steady-state model is considered only. Considering the purpose of this paper is to analyze the behavior of the whole process, the ideal separation model [10] is adopted to facilitate analysis.

$$R=K_AQ_A+K_B(Q-Q_A) \quad (7)$$

$$P=(1-K_A)Q_A+(1-K_B)(Q-Q_A) \quad (8)$$

This model assumes that K_A percent of the coarse particles enter the hydrocyclone underflow returning to the mill to be ground again, while K_B percent of the fine particles enter the hydrocyclone underflow. In common cases, there are conditions: $0\le K_A\le 1$, $0\le K_B\le 1$, and $K_A>K_B$.

When the mill model and the hydrocyclone model are constructed, the transfer function model of the whole closed-circuit grinding-classification process is derived below. The mill model (6) is a nonlinear differential equation and can be linearized in the local area of the steady-state values. Let $\bar{F}$, $\bar{M}$, $\bar{Q}$, $\bar{P}$, $\bar{R}$ denote the steady-state values of the mass flows respectively and similarly use the variables with the overline to denote the steady-state values of the corresponding variables. The real values are denoted by the incremental representation, for example $F=\bar{F}+f$ and $F_A=\bar{F}_A+f_A$ where the lower case letters denote the variation values from the steady-state values. After linearization of equation (6),

$$\frac{dq_A}{dt}-\frac{\bar{Q}_A}{\bar{Q}}\frac{dq}{dt}=\frac{2\bar{Q}\bar{z}_{AM}}{V}q-\frac{\bar{Q}_A}{V}q-\frac{\bar{Q}}{V}q_A-K_Tq_A \quad (9)$$

Based on the two steady-state functions

$$\begin{cases}\bar{M}=\bar{Q}\\ \bar{M}\bar{z}_{AM}-\bar{Q}\bar{z}_{AQ}-K_TV\bar{z}_{AQ}=0\end{cases} \quad \textbf{(10)}$$

we can obtain

$$\bar{z}_{AM}=(K_T\tau+1)\bar{z}_{AQ} \quad (11)$$

where $\tau=V/\bar{Q}$, denoting the average time for the particles to pass through the mill.

In addition, according to the six steady-state functions

$$\begin{cases}\bar{R}=K_A\bar{Q}\bar{z}_{AQ}+K_B(\bar{Q}-\bar{Q}\bar{z}_{AQ})\\ \bar{P}=(1-K_A)\bar{Q}\bar{z}_{AQ}+(1-K_B)(\bar{Q}-\bar{Q}\bar{z}_{AQ})\\ \bar{P}=\bar{F}\\ \bar{M}=\bar{F}+\bar{R}\\ \bar{R}\bar{z}_{AR}=K_A\bar{Q}\bar{z}_{AQ}\\ \bar{M}\bar{z}_{AM}=\bar{R}\bar{z}_{AR}+F\end{cases} \quad \textbf{(12)}$$

we can obtain

$$\bar{z}_{AM}=1-K_B+K_B\bar{z}_{AQ} \quad (13)$$

From equations (11) and (13), we can derive

$$\begin{cases}\bar{z}_{AQ}=\dfrac{1-K_B}{1+K_T\tau-K_B}\\ \bar{z}_{AM}=\dfrac{1+K_T\tau-K_B-K_BK_T\tau}{1+K_T\tau-K_B}\end{cases} \quad \textbf{(14)}$$

Substitute equation (14) into (9) and represent the model by the form of the transfer function to obtain

$$G_A(s)=\frac{q_A(s)}{q(s)}=\frac{1-K_B}{1+K_T\tau-K_B}\frac{s+\dfrac{2K_T\tau+1}{\tau}}{s+\dfrac{K_T\tau+1}{\tau}} \quad (15)$$

The whole closed-circuit grinding-classification process is represented by the block diagram of the transfer functions shown in **figure 3.**

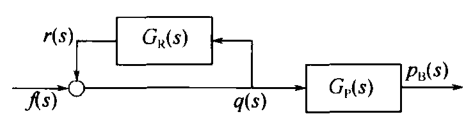

Figure 3 Transfer function representation of the closed-circuit grinding-classification process

In figure 3, $f(s)$, $q(s)$, $r(s)$, $p_B(s)$ denote **the** Laplace transformation of f, q, r, and p_B, respectively. f is the mass flow variation of the raw material and p_B the mass flow variation of the product in the whole process.

$$\begin{cases}G_R(s)=\dfrac{r(s)}{q(s)}=\dfrac{K_Aq(s)+K_B(q(s)-q_A(s))}{q(s)}=(K_A-K_B)G_A(s)+K_B\\ G_P(s)=\dfrac{p_B(s)}{q(s)}=\dfrac{(1-K_B)(q(s)-q_A(s))}{q(s)}=1-K_B-(1-K_B)G_A(s)\end{cases} \quad (16)$$

The transfer function of the whole grinding process is

$$G_C(s)=\frac{p_B(s)}{f(s)}=\frac{G_P(s)}{1-G_R(s)}=\frac{b_1s+b_0}{a_1s+a_0} \quad (17)$$

Substitute equations (15) and (16) into (17) to obtain the parameters of $G_c(s)$:

$$\begin{cases}a_1=K_T\tau^2-K_A\tau+\tau\\ a_0=K_BK_T\tau+2K_T\tau+K_T^2\tau^2+1-K_A-2K_AK_T\tau\\ b_1=K_T\tau^2\\ b_0=K_BK_T\tau+K_T{}^2\tau^2\end{cases} \quad \textbf{(18)}$$

3 Dynamic behavior analysis

The transfer function facilitates the dynamic behavior analysis of the closed-circuit grinding-classification process. What we concern mostly is the time constant of the process. The time constant reflects the speed of dynamic response. In a fist-order transfer function, the time constant equals the reciprocal of the distance from the pole to the origin so the analysis blow concentrates on the pole.

The transfer function $G_C(s)$ has four parameters which are determined by the four physical parameters. K_T and τ are the parameters of the mill while K_A and K_B are the parameters of the hydrocyclone. They are all defined in the section above. $G_c(s)$ is determined not only by the mill but also by the hydrocyclone, illustrating the necessity to analyze the closed-circuit process from the systematic perspective. The pole of $G_c(s)$ is

$$P_C = -\frac{a_0}{a_1} = -\frac{K_B K_T \tau + 2K_T \tau + K_T^2 \tau^2 + 1 - K_A - 2K_A K_T \tau}{K_T \tau^2 - K_A \tau + \tau} \tag{19}$$

The analysis of the pole is able to answer a significant question: "What is the difference between the closed-circuit process and the open-circuit process?" The open-circuit process is equivalent to the closed-circuit process in the special case where $K_A = 0$ and $K_B = 0$. In the case the hydrocyclone is equal to a connector and the coarse particles and fine particles all enter the overflow. The transfer function of the open-circuit process is

$$G_O(s) = \frac{K_T \tau}{1 + K_T \tau} \frac{s + K_T}{s + \dfrac{K_T \tau + 1}{\tau}} \tag{20}$$

The pole of $G_O(s)$ is

$$P_O = -\frac{K_T \tau + 1}{\tau} \tag{21}$$

From equations (19) and (21), it is easy to calculate

$$P_C - P_O = \frac{K_T (K_A - K_B)}{1 + K_T \tau - K_A} > 0 \tag{22}$$

So when an open-circuit process becomes a closed-circuit process, the pole of the process moves towards the origin. At the same time, the time constant becomes large. This is an important difference between the closed-circuit process and the open-circuit process.

Another significant question to be answered by the analysis of the pole is "How do various physical parameters affect the dynamic behavior of the closed-circuit process?" P_c varying with the four physical parameters is shown in **figure 4.**

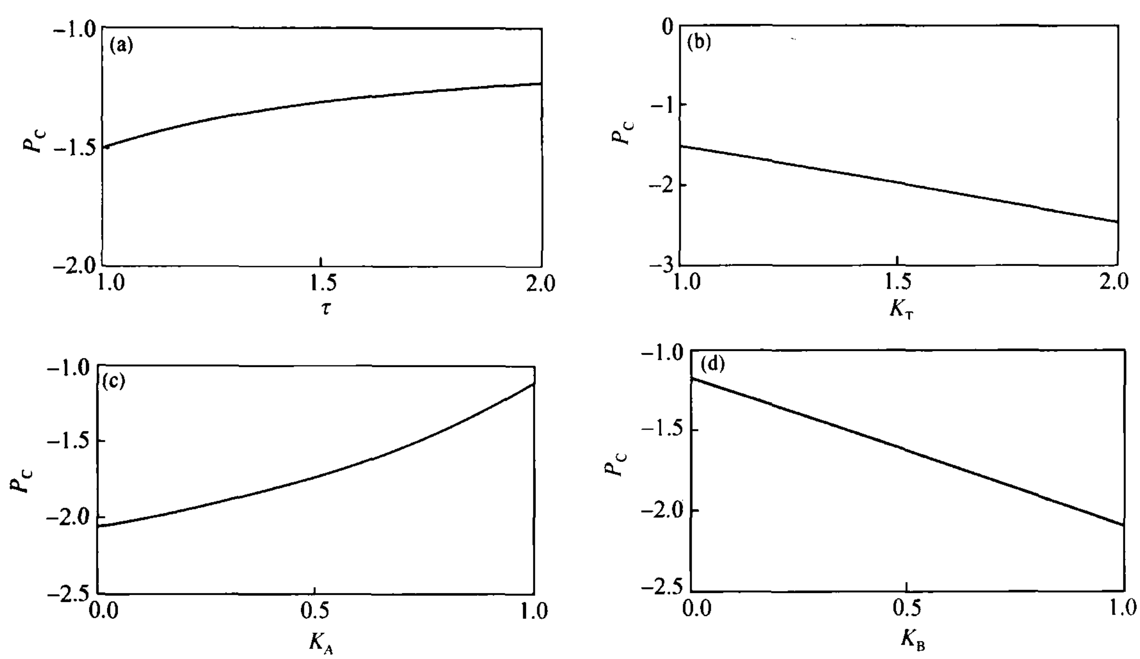

Figure 4 P_c varying with the four physical parameters: (a) τ, (b) K_T; (c) K_A; (d) K_B.

In figures 4(a) and (b), it is easy to see when τ increases or K_T decreases, P_c moves towards the origin. Meanwhile, the time constant of the process grows and the dynamic response becomes slow. Large τ and small K_T reflect the ability of the mill to break particles is weak, and the transformation rate from the coarse particles to the fine particles is small and particles need to stay inside the mill for a long time. In this case particles have to circulate more times in the process, so the time constant is large.

In figures 4(c) and (d), when K_A increases or K_B decreases, P_c moves towards the origin. Meanwhile, the time constant of the process grows and the dynamic response becomes slow. The increase of K_A and the decrease of K_B reflect the separating ability of the hydrocyclone is improved. In the ideal case, where $K_A = 1$ and $K_B = 0$, all the coarse particles enter the underflow and all the fine particles enter the overflow. The hydrocyclone separate the fine particles from the coarse particles completely. In the worst case, where $K_A = 0$ and $K_B=0$, the coarse particles and the fine particles all enter the overflow. The hydrocyclone loses the separating ability and the case is equivalent to the open-circuit grinding process.

4 Conclusions

From the systematic perspective, a first-order transfer function model of the closed-circuit grinding-classification process is built. The input of the model is the mass flow of the rough particles and the output is the mass flow of the fine particles in the hydrocyclone overflow. The model describes the dynamic relation between the raw material and the product. The pole analysis of the transfer function answers two important questions in theory: "What is the difference between the closed-circuit process and the open-circuit process?" and "How do various physical parameters affect the dynamic behavior of the closed-circuit process?" The modeling and analysis of the dynamic behavior of the closed-circuit grinding-classification process provides a systematic method helpful to understand, design and control the process.

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