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3-D distribution of tensile stress in rock specimens for the Brazilian test

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Abstract: It is claimed that the formula used for calculating the tensile strength of a disk-shaped rock specimen in the Brazilian test is not accurate, because the formula is based on the 2-dimensional elastic theory and only suitable for very long or very short cylinders. The Matlab software was used to obtain the 2-dimensional distribution of stress in the rock specimen for Brazilian test. Then the 2-dimensional stress distribution in Brazilian disk was analyzed by the Marc FEM software. It can be found that the results obtained by the two software packages can verify each other. Finally, the 3-dimensional elastic stress in the specimen was calculated. The results demonstrate that the distribution of stress on the cross section of the specimen is similar to that in 2-dimension. However, the value of the stress on the cross section varies along the thickness of the specimen and the stress is bigger when getting closer to the end of the specimen. For the specimen with a height-to-diameter ratio of 1 and a Poisson's ratio of 0.25, the tensile strength calculated with the classical 2-D formula is 23.3% smaller than the real strength. Therefore, the classical 2-D formula is too conservative.

Key words: Brazilian test; tensile strength; FEM; rock; Marc; Matlab

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1 Introduction

Tensile strength, as a factor to measure the characteristics of rock-like materials, has been widely used in the theoretical study of rock mechanics and the application of rock mass engineering.

For general materials, like a metal, tensile strength is measured directly. The specimen is cylinder-shaped and is thicker at the two ends and thinner in the middle. Its working length is 10 or 5 times of its diameter. The shape of the specimen and the loading condition can assure that the uniform tensile force is conducted on the specimen. However, this method is not suitable for rock-like fragile materials. The main reason is that the specimen is hard to prepare, the second reason is that it may cause bias tension on the specimen. So a direct tension test is hard to conduct and the result is not accurate enough.

The most commonly used method for measuring the tensile strength of rock materials is an indirect test, named as the Brazilian test or Split Test. This method uses disk-shaped specimen. A pair of linear load is acted on the disk oriented along the direction of a diameter, see **figure 1.** The specimen will be broken into two semicircles along this diameter during the test. For homogeneous materials, the failure plane is flat with low roughness.

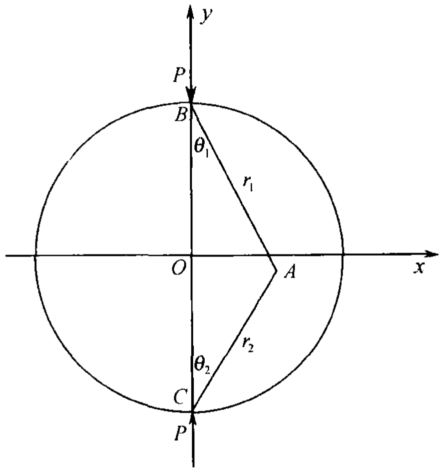

Figure 1 Illustration of the rock disk.

The formula to calculate the tensile strength of the materials in Brazilian test is based on the analytical solution in the 2-D elasticity theory [1]. The forces acting on the disk are illustrated in figure 1, where P represents the loading force and O is the center of the specimen. At an arbitrary point A within the disk, the stress components can be calculated by the following formulas.

$$\sigma_x = \frac{2P}{\pi}\left(\frac{\sin^2\theta_1\cos\theta_1}{r_1}+\frac{\sin^2\theta_2\cos\theta_2}{r_2}\right)-\frac{2P}{\pi d} \tag{1}$$

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$$\sigma_y = \frac{2P}{\pi}\left(\frac{\cos^3\theta_1}{r_1} + \frac{\cos^3\theta_2}{r_2}\right) - \frac{2P}{\pi d} \tag{2}$$

$$\tau_x = \frac{2P}{\pi}\left(\frac{\cos^2\theta_1 \sin\theta_1}{r_1} + \frac{\cos^2\theta_2 \sin\theta_2}{r_2}\right) \tag{3}$$

where the tensile stress is negative while the compression is positive, d is the diameter of the disk. At the ends of the diameter *BC,* there exists stress concentration. However, according to the Saint Venant Principle, the stress concentration can be neglected at the place where is relatively far away from the stress concentration points. When θ_1 and θ_2 are 0 in formula (1), the normal stress along the disc diameter *BC* is tensile stress and evenly distributed with a value of $-2P/(\pi d)$. The minimum compression stress is in the center of the disk and its value is $6P/(\pi d)$ which is 3 times of the tensile stress along *BC.* Since the compressive strength of the rock material is much higher than its tensile strength, the rock specimen is always broken under the tensile stress. This is the mechanical principle for testing the tensile strength of fragile material by using the Brazilian test. If the thickness of the rock disk is t, the force is P, the formula for calculating the tensile strength is:

$$\sigma_T = \frac{2P}{\pi dt} \tag{4}$$

The advantages of the Brazilian test are its simpleness and no need of any special equipment. A general test machine and some simple apparatus are good enough to do the test. Therefore, this method has been world-widely used and has been taken as a standard in many countries, for example, J1SA1113 in Japan, ASTMC-49671 in the United States, and BS1881Part4 in the United Kingdom, *etc.* In China, the Brazilian test has been also included in the authorization documents of Chinese national standard and specialization standards [2-3], which are used to standardize the rock experiment in hydraulic, mining, railway industries and so on. It is even used to test the tensile strength of concrete [4].

Brazilian test has been used for about 40 years to test the tensile strength of rocks. Ever in 1971, American researchers Mellor and Hawkes [5] did profound research on Brazilian method for testing the tensile strength of rocks. Based on their research, the International Society for the Rock Mechanics recommended Brazilian test method as the indirect method for testing the tensile strength of rocks in 1978 [6]. Since that time, China had also adopted the method as a national standard and specialization standard for different industries. For about 40 years, this method has produced a wide and profound influence on many engineering fields.

2 Problems in the Brazilian test

However, people did not pay much attention to the potential problems in Brazilian test. The formula for calculating the tensile strength in Brazilian test is based on the 2-dimensional analytical solutions for a plane stress or a plane strain problem, while the real condition for the test is a 3-dimensional problem. Moreover, the commonly used specimen with a height-to-diameter ratio of 0.5-1.0 [2-3, 6-7] does not guarantee that the loading condition can be simplified as a plane stress problem or a plane strain problem. The loading condition in 3-dimension is much complex than that in 2-dimension, and thus, the stress along the thickness of the specimen varies. So, the formula based on the 2-dimensional analysis is not accurate when used for the 3-dimensional problem. Actually, some researchers already pointed out that the tensile strength measured by the Brazilian test is not equal to the strength measured by the direct testing method for the same materials [8-9]. However, no body doubts the correctness of formula (4) used in the Brazilian test.

To obtain the 3-dimensional theoretical solution of Brazilian test is difficult. Very few researchers are exploring this problem [10]. Since 3-D FEM embraces all the elasticity theories for the spacial problems, it can be considered as a digitized elasticity theory. It can produce a specific solution for a complex problem. This paper will use FEM package Marc to analyze the 3-Dimensional stress distribution in the disk-shaped specimen for Brazilian test.

3 Features of 2-dimensional stress distribution in a Brazilian disk specimen

Assume the diameter of the disk specimen is 50 mm, the force is 1000 N/mm. According to formula (1), we use Matlab to draw the contours of the normal stress σ_x which is in the horizontal direction and shown in **figures 2** and **3.** From those figures, we can see that the stress distribution is symmetric along both horizontal and vertical axis. Figure 2 shows the contours of σ_x, where the center of the specimen has tensile stress while the compressive stress exists around the loading point. Figure 3 shows the contours of the tensile stress that is a part of the contours in figure 2.

We also use FEM software Marc to analyze the 2-dimensional stress distribution in the Brazilian specimen under the plane stress condition. The specimen geometry and the loading force are the same as that used in the Matlab calculation. We use 4-node elements and get 636 elements and 677 nodes. **Figure 4** shows only the contours of the tensile stress in the horizontal direction. As we can see, the stress distributions illustrated in figures 3 and 4 are consistent with each other. The tensile stress σ_x along the diameter in the loading direction is distributed uniformly. So, the result demonstrates that Marc's calculation is consistent with the theoretical solution.

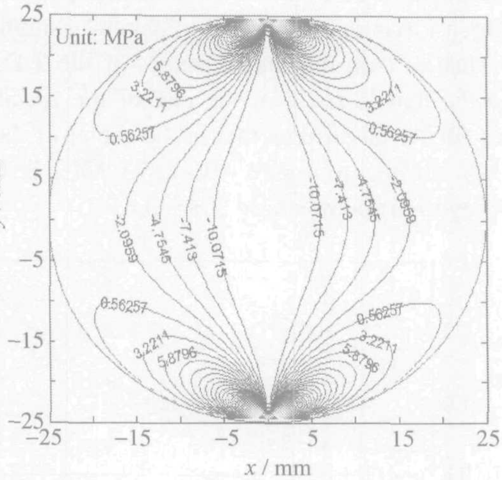

Figure 2 Contours of σ_x by Matlab.

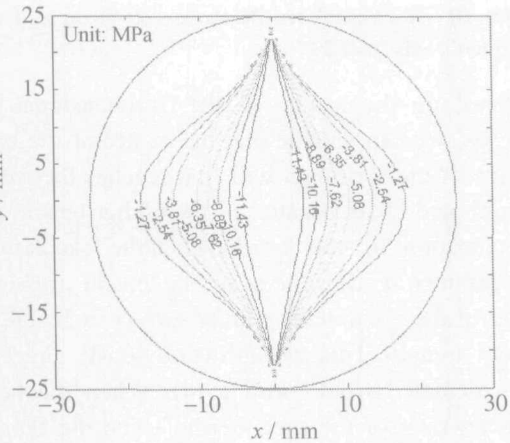

Figure 3 Contours of the tensile stress (σ_x>0) by Matlab.

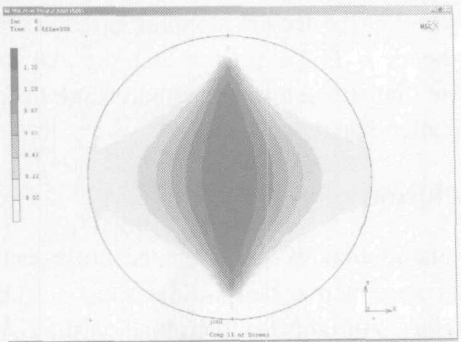

Figure 4 Contours of the tensile stress *(σ_x>0)* by Marc.

4 3-dimensional FEM analysis for the Brazilian Test

4.1 3-D model and material parameters

According to the Chinese national code "Standard for tests method of engineering rock masses (GB/T50266-99)" and "Specifications for rock tests in water conservancy and hydroelectric engineering (SL-264-2001)", the model adopted in our 3D-FEM is 50 mm in diameter and 50 mm in height. Its elastic modulus E is 40 GPa, Poisson ratio μ 0.25, the load P 1000 N/mm, the density ρ 2.7×10^{-3} kg/mm^3. 8-node elements are used and we get 6100 elements. The 3-D FEM model is shown in **figure 5.** For simpleness, we do not benefit from the symmetries of the model.

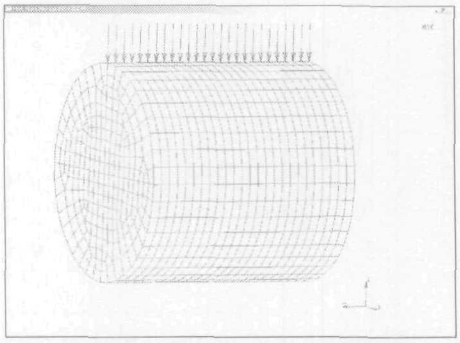

Figure 5 3-Dimensional FEM model.

4.2 Stress distribution on different cross sections

The elasticity analysis was conduct based on the model in figure **5. Figure 6** is the contour bands of the horizontal normal stress σ_x on a cross section of the specimen. It shows only the tensile stress. From figure 6 we can see that the stress distribution in 3-Dimension is very similar to that in 2-Dimension. The pattern of contour lines of σ_x still looks like a flower bud. However, the values of the stress on different cross sections are different. **Figure 7** shows the contour bands of σ_x on an end surface and a cross section in the middle of the specimen. Obviously, the value of σ_x on the end surface is larger than that on the cross-section at the middle.

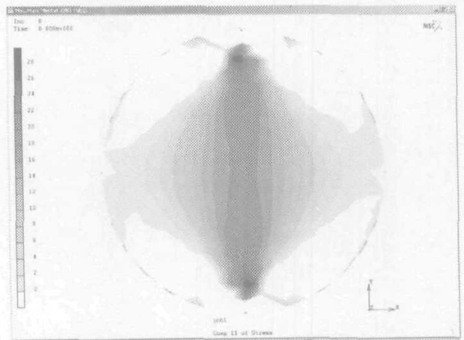

Figure 6 Contour bands of the horizontal tensile stress on a cross section.

4.3 Stress distribution on the specimen's meridian planes

A section including the axis of the cylinder is called a meridian plane. To investigate the stress changes in specimen, we get contour band graphs of σ_x on the vertical meridian plane and the horizontal meridian plane of the specimen, as shown in **figures** 8 and 9. From these graphs, we can see that the stress distribution in the specimen is symmetric to the axis of the specimen. Moreover, it can be seen that σ_x varies along the thickness of the specimen: the horizontal tensile stress is bigger when it is closer to the end of the specimen.

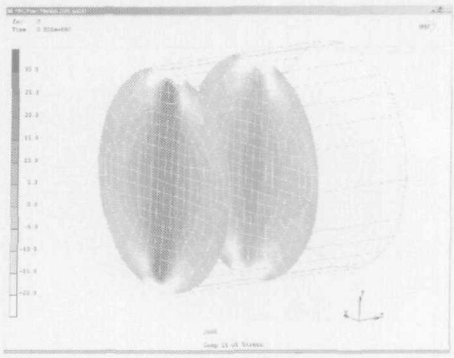

Figure 7 Comparison of σ_x on the surface at the end and the middle of the specimen.

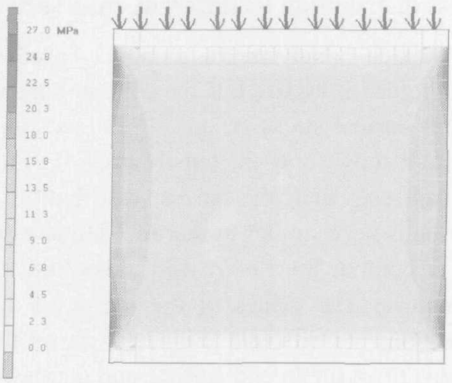

Figure 8 Contour bands of σ_x on the vertical meridian plane of the specimen.

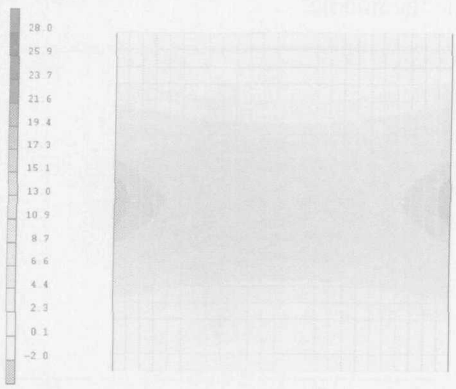

Figure 9 Contour bands of σ_x on the horizontal meridian plane of the specimen.

Figure 10 is the variety of the horizontal tensile stress σ_x along the axis of the specimen. The graph displays only half of the curve because of symmetries. The value on the x-axis is a relative thickness. The origin of the coordination is at the central point of the specimen's axis. The coordinate x of 1 is corresponding to the end point of the axis. The y-axis represents the relative stress which is from σ_x divided by σ_0 (12.73 MPa). σ_0 is the tensile stress in the center of the 2-D disk in figure 3. From the FEM results, it can be seen that the tensile stress at the central point of the specimen's axis under the 3-dimensional condition is 13% smaller than that in the center of the 2-D disk, while the tensile stress at the end of the specimen's axis is 30.4% larger than that in the center of the 2-D disk. According to figure 10, 12.73 MPa is corresponding to the point whose z/H is 0.65.

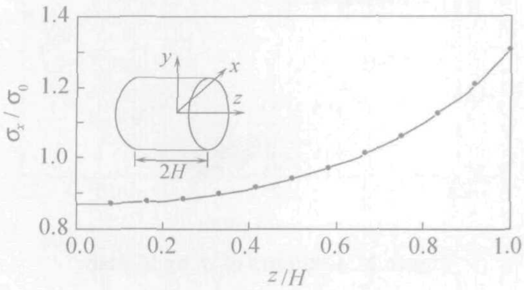

Figure 10 Horizontal tensile stress $(\sigma_x>0)$ on half of the specimen's axis (μ=0.25).

Based on the results of the 3-dimensional FEM analysis, we can deduce that the center of the ending surface of the specimen will first reaches their tensile strength and break because of the higher tensile stress at its location. In other words, when the load acting on the specimen reaches the peak, the tensile stress at the center of the specimen's ending surface is actually the tensile strength. This strength is obviously larger than that calculated from formula (4). When the height-diameter ratio of the specimen is 1 and the Poisson's ratio is 0.25, as calculated in this paper, the tensile strength is 30.4% larger than that obtained under 2-dimension, *i.e.* the tensile strength based on the 2-D elastic theory is 23.3% smaller than the real strength. Therefore, the strength obtained according to formula (4) is smaller than the real value.

5 Conclusion

The distribution of stress on the cross-section of Brazilian specimen under 3-dimension is similar to that obtained from the 2-dimensional elasticity theory. The pattern of contour lines for the horizontal tensile stress looks like a flower bud. The value of horizontal tensile stress varies along the thickness of the specimen. The tensile stress at the end of the specimen is larger than that in the middle of the specimen. For the Brazilian disk specimen with a height-diameter ratio of 1 and a Poisson's ratio of 0.25, the tensile strength is 30.4% larger than that obtained under 2-dimension. This demonstrates that the tensile strength calculated with formula (4) is 23.3% smaller than the actual strength. From the engineering point of view, the tensile strength calculated from formula (4) is too safe or too conservative.

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