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Drag characteristics of power law fluids on an upstream moving surface

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Abstract: The specific problem to be considered here concerns the boundary layer problem of a non-Newtonian fluid on a flat plate in length, whose surface has a constant velocity opposite in the direction to that of the mainstream with $U_W >> U_\infty$, or alternatively when the plate surface velocity is kept fixed but the stream speed is reduced to zero. A theoretical analysis for a boundary layer flow is made and the self-similar equation is determined. Solutions are presented numerically for special power index and the associated transfer behavior is discussed.

Key words: boundary layer theory; power law fluid; similarity solutions

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1 Introduction

When a fluid flows past a solid body at high Reynolds number, a thin viscous boundary layer is known to form at least along the forward portion of the solid surface [1-2]. Under the influence of even a mild adverse pressure gradient, however, the motion within this boundary layer may be retarded to the extent that beyond a certain point, the direction of the flow near the surface becomes reversed. When this occurs, the forward moving fluid in the boundary detaches from the surface at the point, being deflected away from the wall by a region of back-flow which forms downstream. This phenomenon is commonly known as separation. Klemp and Acrivos [3-4] studied the momentum boundary layer problem induced by a flat plate moving in the direction opposite to that of the mainstream at high Reynolds numbers. A new technique for numerically integrating the boundary layer equations through the region of reverse flow, which takes downstream influence into account, was presented, since the standardization technique does not work. In their studies, they assumed that a region of reverse flow remains confined within a boundary layer and the conventional boundary layer equations should continue to apply down-stream of the point of detachment. In the upstream portion of the separated region, the boundary layer equation has a similar solution. Later Casal [5], Черный [6-7], Hussaini *et al.* [8-9], Vajravelu and Mohapatra [10], Soewono *et al.* [11] showed that the solutions of such boundary layer problems do not exist for the values of velocity ratio parameter X larger than a positive critical value λ^*. The existence of solutions is proved for $0<\lambda\le\lambda^*$ by considering an equivalent initial-value problem. However, for $0<\lambda<\lambda^*$, the solutions of the boundary value problem are found to be non-unique.

Recently, Akcay *et al.* [12] studied the drag reduction of a non-Newtonian fluid by fluid injection on an upstream moving wall. Zheng *et al.* [13-14] discussed the power law fluid boundary layer equations and the nonlinear boundary value problems corresponding to the surface moving opposite to the direction of the main stream. By introduce the similarity transformation group, they reduced the boundary layer equation into a singular nonlinear two-boundary value problem. Sufficient conditions for existence, non-uniqueness, and analyticity of positive solutions to the problems were established utilizing the perturbation and shooting techniques. They showed that the solutions of this boundary layer is crucially dependent on the parameters of the power law index n and the velocity ratio λ, and there exists a critical range of values of those parameters such that the boundary layer differential equations admit analytical solutions. The solutions of

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such boundary layer problems do not exist for the parameters overrun the critical range. However, for the parameters in the critical range, solutions of the boundary value problem are found to be non-unique.

This failure to obtain solutions to the boundary layer equations for overrun the critical range could be explained, by supposing that viscous effects are no longer confined to a thin, $O(R^{-\frac{1}{N+1}})$ layer adjacent to the plate. Nevertheless, we do recall that when the velocity ratio $\lambda \to \infty$, or alternatively when the plate surface velocity is kept fixed but the stream speed is reduced to zero, a solution of boundary layer type still exists in which the boundary layer grows upstream, starting from the trailing edge. Consequently it seems reasonable to suppose that the boundary layer simplifications would apply for intermediate values of the velocity ratio λ given for specified parameters of the power law index that they are known to remain valid for the velocity ratio does not go beyond its critical value and $\lambda >> 1$.

The specific problem to be considered here concerns the boundary layer problem of a non-Newtonian fluid on a flat plate in length, whose surface has a constant velocity opposite in the direction to that of the mainstream with $U_W >> U_\infty$. We assume that when the velocity $U_W \to +\infty$, or alternatively when the plate surface velocity is kept fixed but the stream speed is reduced to zero, the boundary layer still exists in which the boundary layer grows upstream, starting from the trailing edge. Let the X axes be taken along the plat starting from the trailing edge, and Y axes be normal to it, U and V are the velocity components parallel and normal to the plate, with positive X, U being the direction of the flat plate velocity (opposite to the direction of **figure 1)**, by introducing the following dimensionless variables:

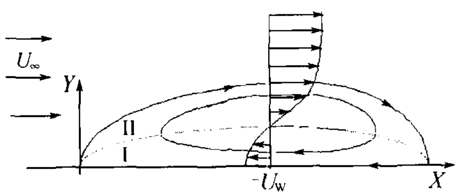

Figure 1 Uniform flow past a finite flat plate with a surface velocity $-U_w$.

$$x = X/L,\ y = \left[\rho U_W^{2-n} L^n / K\right]^{\frac{1}{n+1}} Y/L,\ u = U/U_W,$$
$$v = \left[\rho U_W^{2-n} L^n / K\right]^{\frac{1}{n+1}} V/U_W,\ N_{Re} = (U_W^{2-n} L^n)/\gamma \qquad (1)$$

we obtained the boundary layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) \qquad (3)$$

The boundary conditions are

$$u\big|_{y=0} = 1,\quad v\big|_{y=0} = 0,\quad u\big|_{y=+\infty} = 0 \qquad (4)$$

2 Two-point boundary value problems

The stream function $\psi\ (x, y)$ and similarity variable η are defined:

$$\psi = Ax^{\alpha} f(\eta),\quad \eta = Bx^{\beta} y \qquad (5)$$

where A, B, α and β are the constants to be determined, and $f(\eta)$ denotes the dimensionless stream function. Thus, the u velocity components is:

$$u = \frac{\partial \psi}{\partial Y} = ABx^{\alpha+\beta} f'(\eta) \qquad (6)$$

Choosing $\beta = -\alpha$, $AB=1$, then

$$v = -\frac{\partial \psi}{\partial x} = -A\alpha\, x^{\alpha-1}[f(\eta) - \eta f'(\eta)] \qquad (7)$$

Equation (2) is satisfied automatically. Substituting u and v defined by equations (6) and (7) into (3) and (4) combining with $\alpha = 1/(n+1)$, $B = (n+1)^{-1/(n+1)}$, we obtain

$$(|f''(\eta)|^{n-1} f''(\eta))' + f(\eta) f''(\eta) = 0 \qquad (8)$$

$$f(0) = 0,\ f'(0) = 1,\ f'(\eta)\big|_{\eta=+\infty} = 0 \qquad (9)$$

We assume that the solution of equations (8) and (9) possesses a negative second derivative $f''(\eta)$ in $(0, +\infty)$ and $f''(+\infty) = 0$ (where $f''(\eta) < 0$ in $(0, +\infty)$ is closely related boundary condition). Defining the variable transformation as follows:

$$g(t) := [-f''(\psi(x))]^n,\ t = f(\eta),\ t \in [0, 1) \qquad (10)$$

where t is the dimensionless tangential velocity, $g(t)$ the dimensionless shear force. Substituting equation (10) into (8) and (9) and applying the chain rule yields the following singular nonlinear two-point boundary value problems:

$$g''(t) = -t g^{-1/n}(t),\quad 0 < t < 1 \qquad (11)$$

$$g(0) = 0,\quad g'(1) = 0 \qquad (12)$$

In the above two-point boundary value problems, the tangential velocity t is taken as the independent variable and the shear stress $g(t)$ as the dependent variable, the so-called Crocco variable. It may be seen from the derivation process only the positive solutions of equations (11) and (12) are physically significance.

3 Solutions and discussion

The singular nonlinear two point boundary values (11) and (12) are solved for the dependent variable g as a function of t for various parameters of n by using the shooting technique.

The technique is called a "shooting" method, by analogy to the procedure of firing objects at a stationary target. We express the boundary and equation at point $(t_i, g(t_i))$ by

$$g_1 = g_0 + C \cdot \Delta t \text{ , } (g_{i+1} - 2g_i + g_{i-1})/(\Delta t)^2 = -t_i / g_i^{-1/n} \text{ ,}$$

where $\Delta t = 1/m$. We start with a parameter σ_0 ($\sigma_0 = g(1)$) that determines the initial elevation (or reduce) at which the object is fired from the point $(t_0, g(t_0))$ and along the curve described by the solution to the initial-value problem. If $g(0,\sigma_0)$ is not sufficiently close to zero, we correct our approximation by choosing new values σ_1, σ_2 and so on, until $g(0,\sigma_k)$ is sufficiently close to "hitting" the point zero.

Figure 2 shows that when the skin friction decreases with the increase of n the function $g(t)$ has a positive local maximum point at t = 1. The physical meaning indicates that the slug-like profiles exhibited by a small n power law fluids exert a greater shear stress and has a greater thrust force along the plate, and the boundary layer is thinner in the case of small power law index.

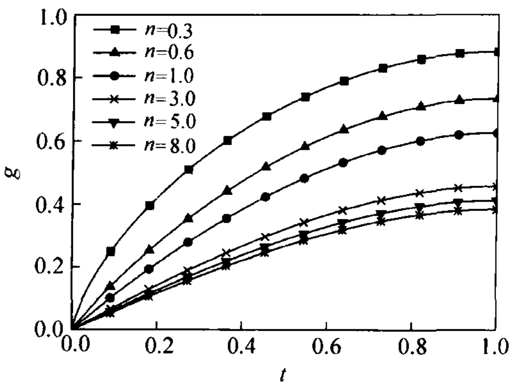

Figure 2 Shear stress profiles (g) for n=0.3 to 8.0.

4 Conclusion

The theoretical problem of external, boundary layer flow of a non-Newtonian fluid is made and the momentum transfer characteristic on an upstream continuous moving wall in power law fluids with $U_W >> U_\infty$ is discussed. The full similarity equations and the types of potential lows necessary for similar solutions to the boundary layer equations are established, the solutions are presented for special power law index and the shear stress behavior is discussed.

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