

## A mathematical model capable of describing the liquid flow mainly in a blast furnace

Cheng-shan Wang<sup>1)</sup> and Xiao-jing Mu<sup>2)</sup>

1) College of Materials Science and Engineering, Chongqing University, Chongqing 400045, China

2) College of Chemistry and Chemical Engineering, Chongqing University, Chongqing 400045, China

(Received 2008-10-21)

**Abstract:** The molten liquid flow inside a packed bed is a familiar momentum transportation phenomenon in a blast furnace. With regard to the reported mathematical models describing the liquid flow within a packed bed, there are some obstacles for their application in engineering design, or some limitations in the model itself. To overcome these problems, the forces from the packed bed to the liquid flow were divided into appropriate body and surface forces on the basis of three assumptions. Consequently, a new mathematical model was built to present the liquid flow inside the coke bed in a blast furnace. The mathematical model can predict the distribution of liquid flowrate and the liquid flowing range inside the packed bed at any time. The predicted results of this model accord well with the experimental data. The model will be applied considerably better in the simulation on the ironmaking process compared with the existent models.

**Key words:** ironmaking process; blast furnace; packed bed; discrete flow; mathematical model

[This work was financially supported by the National Natural Science Foundation of China (No.50704040, 20805060) and the Natural Science Foundation Project of Chongqing Science & Technology Commission, China (No.CSTC,2009BB4197).]

### 1. Introduction

The molten liquid flow inside a packed bed is a typical momentum transportation phenomenon in the dropping zone of a blast furnace. The study on the liquid discrete flow inside the packed bed will give a theoretical base for mathematical simulation of the ironmaking process, and therefore, give guidance for the development of the ironmaking process. As to the reported mathematical models describing the liquid flow within the blast furnace, there are some barriers for their application to engineering scale-up or some limitations in the mathematical descriptions.

The first kind of model, taking the continuum approach [1-6], determines the components of liquid velocity and the liquid pressure using the equations of continuity and motion. The equations used in this approach were solved in the domain, which was the liquid flowing zone in the packed bed, but not the whole bed zone where all particles were packed. That is to say, the continuum approach could not predict the li-

quid flowing extent inside the packed bed.

Another kind of model, which may be called the structure model [7-10], takes into account the specific structure of the packed bed. It can predict the liquid flowing extent inside the packed bed; however, the numerical grids for the solution of transportation equations coupled with this kind of model should correspond to the size of packed particles, and this will increase the computational work considerably [11-12].

The third kind of model, the so-called force balance model [13-15], in which three forces of gas drag force acting on liquid droplets, the gravity of liquid droplets, and the resistance force from bed particles to liquid are balanced, is unable to describe the dispersal phenomenon of a liquid stream starting from a point in the packed bed.

In the fourth kind of model, which may be called the probability model [11,16-17], the dispersal phenomenon of a liquid stream starting from a point in the packed bed was described using a stochastic velocity

variable and its distribution function. However, this kind of model has not yet given the prediction of the non-steady liquid flow inside the bed after liquid stream is introduced to the packed bed. As a model parameter, the maximum of stochastic velocity variable was determined by the slope of a tangent of the experimental curve in the probability model. There should be enough experimental data points beside the tangent point in the experimental curve. However, in this model, there was only an experimental point beside the tangent point [16], and the other data points in the experimental curve had little contribution to the determination of the model parameter.

The purpose of this study is to produce an alternative mathematical model that can be used to simulate liquid flows within the coke-packed bed in a blast furnace. This model will have a considerably more complete mathematical description of liquid flow inside the packed bed, in contrast to the force balance model and the probability model, and it can predict the distribution of liquid flowrate, and especially the liquid flowing extent inside the packed bed, which cannot be realized by the continuum approach. The numerical solution of this model may be performed in the same grid system as used in the numerical solution for general transportation equations, and is not needed to carry out in the numerical grid corresponding to the size of packed particles any more.

## 2. Mathematical model

Owing to the non-wetting feature between molten liquid and coke particles, the molten liquid flows as droplets inside the bed packed by fuel particles [11, 13]. This liquid discrete flow can be represented using several variables, *i.e.* the interstitial velocity,  $\mathbf{u}_1 = (u_{1x}, u_{1y})$ , the volume fraction of flowing liquid,  $\varepsilon_1$ , and the diameter of liquid droplet,  $d_l$ .

### 2.1. Assumptions used in the model

This model uses the following three assumptions.

(1) The direction of the bed resistance force,  $\mathbf{f}_r$ , is opposite to the direction of liquid flow, and this may be expressed as

$$\mathbf{f}_r = -R\mathbf{u}_1 \quad (1)$$

where

$$R = k_R \varepsilon_1 \quad (2)$$

$k_R$  may be determined by experiments, and in this study,  $k_R$  is obtained from Ref. [2]:

$$k_R = \frac{180(1-\varepsilon)^2 \mu_1}{\varepsilon^3 d_p^2} \quad (3)$$

where  $\varepsilon$  is the voidage of the packed bed,  $\mu_1$  the liquid viscosity, and  $d_p$  the diameter of the packed particle.

(2) There is no surface force on the control surface perpendicular to the direction of the liquid velocity,  $\mathbf{u}_1$ .

(3) The normal stress,  $p_{nn}$ , on the control surface parallel to the direction of liquid flow, is proportional to the volume fraction of the flowing liquid in bed:

$$p_{nn} = k\varepsilon_1 \quad (4)$$

where  $k$  is a model parameter and may be determined by experiment,  $\mathbf{n}$  the unit vector perpendicular to the control surface, which is parallel to the direction of the liquid velocity  $\mathbf{u}_1$ ;  $\mathbf{u}_1 \cdot \mathbf{n} = 0$ .

The force balance model uses the first assumption only to describe the bed actions to the flowing liquid, and the description is not complete. As illustrated in Fig. 1, a liquid stream is introduced at a point on the top of the packed bed, and subsequently flows downwards owing to the action of gravity. The stream is composed of several liquid droplets and is dispersed. After the liquid flow in the bed achieves steady state, the breadth of the lower part of the stream is larger than that of the upper part of the stream. In this case, the direction of the initial liquid flow caused by the liquid gravity is vertically downward. If only use the first assumption, the direction of the liquid flow and that of the bed resistance force are in the same plumb line, that is to say, the liquid stream cannot be dispersed. The bed resistance force expressed by the first assumption and the liquid gravity are two body forces that act on the inside of the control volume. They cannot be used to completely describe the dispersal phenomenon of the liquid stream illustrated in Fig. 1. The forces from the packed bed to the flowing liquid should include not only the body force but also the surface force acting on the surfaces of the control volume. The surface forces are presented by the second and the third assumptions.

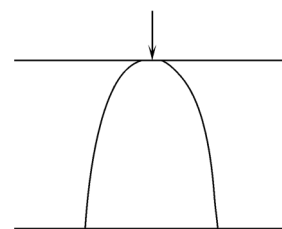


Fig. 1. Dispersal phenomenon of a liquid stream in a packed bed.

The second assumption arises from the phenomenon illustrated in Fig. 2. The forces causing the

movement of discrete liquid droplets inside the packed bed is the gravity of the liquid droplets and (or) the gas drag force to the liquid droplets, rather than the liquid pressure. As designated in Fig. 2, while all liquid droplets are flowing together in a stream line, liquid droplets do not contact the liquid droplets in front of or behind them. This explains that liquid droplets do not receive the action from the liquid droplets in front of or behind them.



Fig. 2. Liquid droplets in a stream without contacting each other.

Based on the reciprocal theorem of shearing stress ( $p_{xy}=p_{yx}$ ) and the second and the third assumptions, it is known that there is no shear stress on the control surface parallel to the direction of the liquid flow. The second and the third assumptions express the stress state at any position. The third assumption comes from the analysis of the liquid flow in the packed bed. If the flowing liquid is composed of several droplets, which have the same size, the liquid volume fraction reflects the amount of the droplets in a unit bed volume. When the liquid is composed of several droplets flowing forwards in the packed bed, the liquid droplets will disperse on the face perpendicular to the direction of the liquid flow. The larger the amount of liquid droplets, the more obvious is this dispersal phenomenon.

2.2. Basic equations of the model

In the  $x', y'$  rectangular coordinate system (see Fig. 3, the direction of the liquid flow points to the same direction of  $x'$  axis), based on the above assumptions and using  $p_{x'x'}$ ,  $p_{y'y'}$ , and  $p_{x'y'}$ , the stress state is expressed as

$$p_{x'x'} = 0 \tag{5}$$

$$p_{y'y'} = k\varepsilon_1 \tag{6}$$

$$p_{x'y'} = 0 \tag{7}$$

By stress analysis [18] and using  $p_{xx}$ ,  $p_{yy}$ , and  $p_{xy}$  in the  $x, y$  rectangular coordinate system (see Fig. 3), the stress state is written as

$$p_{xx} = k\varepsilon_1 \frac{u_{ly}^2}{u_{lx}^2 + u_{ly}^2} \tag{8}$$

$$p_{yy} = k\varepsilon_1 \frac{u_{lx}^2}{u_{lx}^2 + u_{ly}^2} \tag{9}$$

$$p_{xy} = k\varepsilon_1 \frac{-u_{lx}u_{ly}}{u_{lx}^2 + u_{ly}^2} \tag{10}$$

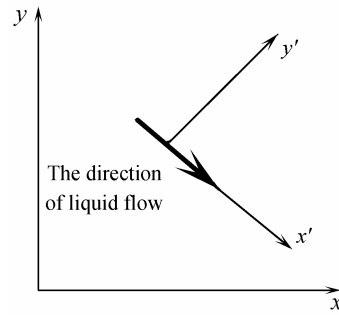


Fig. 3. Position relation of  $u_1$ , the  $x', y'$  coordinate system, and the  $x, y$  coordinate system.

A control micro-volume  $dx \cdot dy$  is selected in the  $x, y$  rectangular coordinate system (see Fig. 4). When the spatial position changes from  $(x, y)$  to  $(x+dx, y+dy)$ , all components of the stress change from  $p_{xx}, p_{xy}, p_{yy}, p_{yx}$  to  $p_{xx}+dp_{xx}, p_{xy}+dp_{xy}, p_{yy}+dp_{yy}$ , and  $p_{yx}+dp_{yx}$ . Based on the conservation of momentum in the  $x$  or  $y$  direction, the following equations are obtained:

$$-\frac{\partial p_{xx}}{\partial x} - \frac{\partial p_{yx}}{\partial y} - k_R \varepsilon_1 \cdot u_{lx} + \rho_l \varepsilon_1 f_x = 0 \tag{11}$$

$$-\frac{\partial p_{xy}}{\partial x} - \frac{\partial p_{yy}}{\partial y} - k_R \varepsilon_1 \cdot u_{ly} + \rho_l \varepsilon_1 f_y = 0 \tag{12}$$

where  $\rho_l$  is the liquid density, and  $f_x, f_y$  are respectively the  $x$ -axis and  $y$ -axis components of the resultant force ( $f$ ) of gas drag force to liquid droplets and liquid gravity.

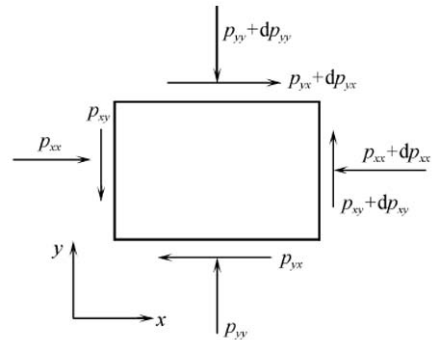


Fig. 4. Stress components on control surfaces.

The action of the packed bed to flowing liquid (including surface force and bed resistance force acting on the inner of control volume) and the body forces such as the gravity of liquid droplets and the gas drag force to liquid are considered in the above momentum conservation equations. It is accepted by most researchers that velocities of the molten liquid within the packed bed in a blast furnace are mainly determined by body forces, such as liquid gravity and gas drag force to liquid droplets, and thus the inertial force may be omitted [11, 13-17, 19]. Therefore, the above

equations do not include the item of inertial force.

By use of the same control micro-volume  $dx \cdot dy$  in Fig. 4 and based on the conservation of liquid mass, the following equation can be obtained:

$$\begin{cases} -\frac{\partial}{\partial x} \left( k\varepsilon_1 \frac{u_{ly}^2}{u_{lx}^2 + u_{ly}^2} \right) - \frac{\partial}{\partial y} \left( k\varepsilon_1 \frac{-u_{lx}u_{ly}}{u_{lx}^2 + u_{ly}^2} \right) - k_R \cdot \varepsilon_1 u_{1x} + \rho_1 \varepsilon_1 f_x = 0 \\ -\frac{\partial}{\partial x} \left( k\varepsilon_1 \frac{-u_{lx}u_{ly}}{u_{lx}^2 + u_{ly}^2} \right) - \frac{\partial}{\partial y} \left( k\varepsilon_1 \frac{u_{lx}^2}{u_{lx}^2 + u_{ly}^2} \right) - k_R \cdot \varepsilon_1 u_{1y} + \rho_1 \varepsilon_1 f_y = 0 \\ \frac{\partial(\varepsilon_1 u_{lx})}{\partial x} + \frac{\partial(\varepsilon_1 u_{ly})}{\partial y} + \frac{\partial \varepsilon_1}{\partial t} = 0 \end{cases} \quad (14)$$

The resultant force acting on the flowing liquid in unit control volume,  $f$ , is

$$f = f_g + f_d \quad (15)$$

where

$$f_g = \varepsilon_1 \rho_1 g \quad (16)$$

$$f_d = \frac{6\varepsilon_1}{\pi d_1^3} \cdot C_d \cdot \left( \frac{\pi}{4} d_1^2 \right) \cdot \left( \frac{1}{2} \rho_g |u_g| \right) \cdot u_g \quad (17)$$

where  $f_g$  is the liquid gravity in unit control volume,  $f_d$  the gas drag force to liquid droplets in unit control volume [2, 7-11, 13-17, 19],  $C_d$  the drag coefficient,  $\rho_g$  the gas density,  $u_g$  the gas interstitial velocity in bed; and  $g$  the acceleration of gravity.

It was validated by experiments [19] that the diameter of the non-wetting liquid droplet inside the blast furnace ( $d_1$ ) varied little when the viscosity and composition of liquid and gas, the liquid flowrate, and the gas velocity in bed varied, and  $d_1$  was mainly determined by the packed bed structure. Here, let the diameter of the liquid droplet  $d_1 = \frac{2}{3} \frac{\varepsilon}{1-\varepsilon} d_p$  [2, 13].

Generally, the basic equations of this model are solved numerically. The detailed numerical method for this model can be found according to our work elsewhere [12].

### 3. Validation of the model

Sugiyama [2] performed the experiment to measure the distribution of liquid flowrate in a tridimensional packed bed where there is no gas flow. In his test, the inlet liquid stream was introduced at a point on the top surface of the tridimensional bed, and the liquid used for the experiment was a mixture of water and glycerin. The present mathematical model was used to simulate Sugiyama's test. Fig. 5 shows the observed and calculated distribution of liquid mass flowrate in

$$\frac{\partial(\varepsilon_1 u_{lx})}{\partial x} + \frac{\partial(\varepsilon_1 u_{ly})}{\partial y} + \frac{\partial \varepsilon_1}{\partial t} = 0 \quad (13)$$

Finally, the stress components in Eqs. (11) and (12) are replaced by Eqs. (8)-(10), and the base equations of this mathematical model are obtained as follows.

steady state at different dropping heights in the packed bed with particles of 6 mm in diameter. During the calculation by this model, the value of  $k$  is  $2.41 \text{ g} \cdot \text{cm}^{-1} \cdot \text{s}^{-2}$ . In Fig. 5, the area below the curve calculated by the present model is equal for all distributions at differential dropping heights, since the liquid flow reaches steady state and should keep mass conservation. It may be concluded that the measured value of Sugiyama has an error, because the measured points move from above the curve calculated by the present model to below it with the increase in the liquid dropping height. Taking no account of the effect of the error, it can be concluded that the calculated result of the present model accords well with the observations.

The liquid flow phenomenon in the packed bed with a horizontal gas flow observed by Ohno [7] was also simulated using the present mathematical model. In Ohno's measurement, the liquid stream is introduced at a point on the top of the bed, and the particle-packed apparatus for the experiment is a box of 54 cm high, 100 cm long, and 5 cm wide (internal). The packed particles are glass spheres with the diameter of 10 mm. Ohno's experiment is a cold physical model test, where the liquid flow condition is mainly similar to that inside the blast furnace. Generally, the flowing liquid volume fraction anywhere in the packed bed is considerably smaller than the voidage of the bed [2], therefore, the liquid in bed has almost no effect on the gas flow in the packed bed. The horizontal gas flow introduced in bed is also a uniform flow in Ohno's experiment. The gas flow velocity ( $u_g$ ) in bed is figured out by the gas blown in and the bed voidage.

Fig. 6 shows the comparison between the observed liquid flowing zone of Ohno [7] and the prediction of the present model. During the computation, model parameter  $k=1.28 \text{ g} \cdot \text{cm}^{-1} \cdot \text{s}^{-2}$ , and  $C_d=0.44$  [7]. In Fig. 6, the predicted boundary line of the liquid flowing zone in steady state by the present model is an

iso-value curve of  $10^{-5} \text{ kg}\cdot\text{cm}^{-2}\cdot\text{min}^{-1}$ , and the liquid flowrate inside the predicted boundary is larger than 99.5% of the total liquid flowrate at the inlet. It is seen that the predicted results have good agreements with the observations. The liquid flowing zone will enlarge when the horizontal gas flowrate is increased. When the horizontal gas flowrate is increased, the angle between the liquid flow direction and the vertical direction will become larger; thus, the liquid flow path and the residence time of the flowing liquid will both lengthen, and finally the liquid flowing zone will enlarge.

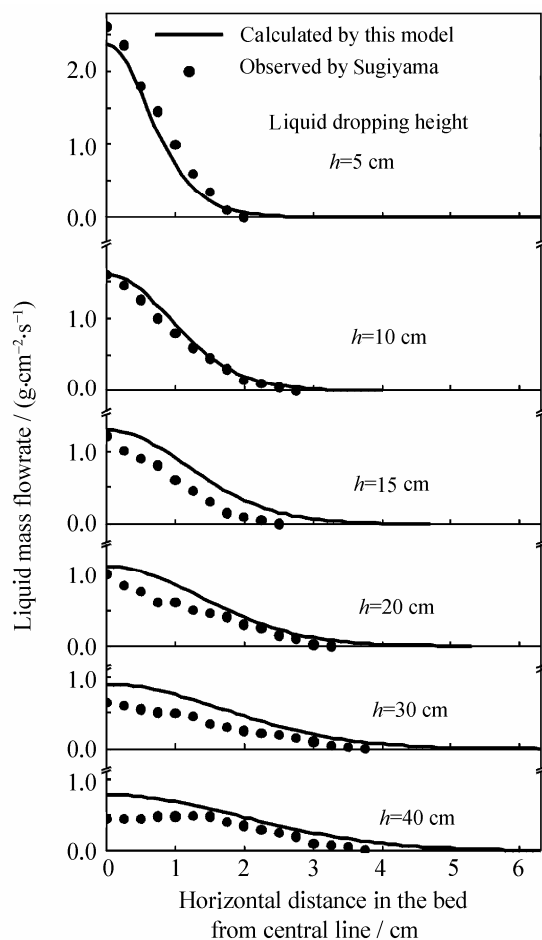


Fig. 5. Comparison of the calculated results with Sugiyama's experimental results.

#### 4. Imaginable application of the model to the ironmaking process

Owing to the development of ironmaking technology, mathematical simulation on the blast furnace process has not yet stopped [1, 3-6]. In blast furnace simulation, the continuum theory, such as potential theory or Navier-Stokes equation, is mainly adopted to present the liquid flow inside the coke-packed bed. The advantages for adopting the continuum theory are that the equations used by the continuum theory and other transportation equations coupled with them have

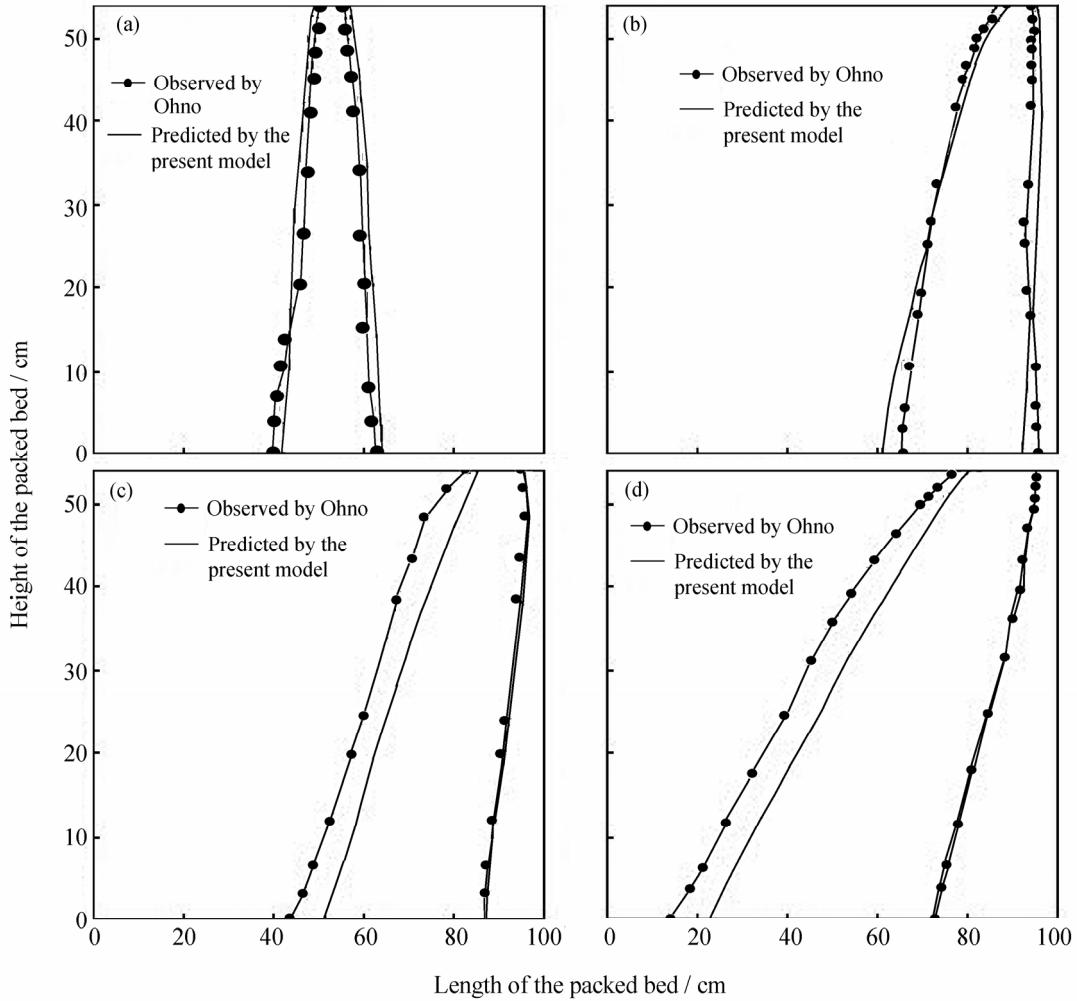
an identical numerical method for solving all of them, and the grid length for numerical calculation is required to obtain good accuracy of numerical solution and is not needed to reduce to the size of packed particles. Without these advantages and also having some limitation in mathematical description, the structure model has not yet been used in the simulation on the actual blast furnace process.

However, there are some limitations in the presentation of the liquid flow inside the coke bed within the blast furnace using the potential theory or the N-S equation. At any time, to predict the distribution of the liquid flowrate inside the bed, the liquid flowing zone inside the bed should be measured first, and then the equations used by the continuum approach should be solved within this measured zone, and thus, the simulated results have reasonably good agreements with the observations. The variation in the distribution of liquid flowrate with time may be regarded as an experimental result in nature, not a predicted result by a mathematical model. We can say that the liquid flowing zone inside the coke bed within the blast furnace cannot be predicted by the potential theory or the N-S equation. The present mathematical model is solved within the entire packed bed zone, and can give the prediction of the liquid flowrate distribution inside the bed and its variation with time. As a result predicted by the present model, Fig. 7 shows the calculated liquid flowing zone (gray-colored) inside the packed bed having five point-liquid-stream sources, at 0.05, 0.5, 5, 10, and 60 s, respectively, after the five point-liquid-stream sources are introduced synchronously. The boundary line of the liquid flowing zone is an iso-flowrate curve of  $10^{-5} \text{ kg}\cdot\text{cm}^{-2}\cdot\text{min}^{-1}$ .

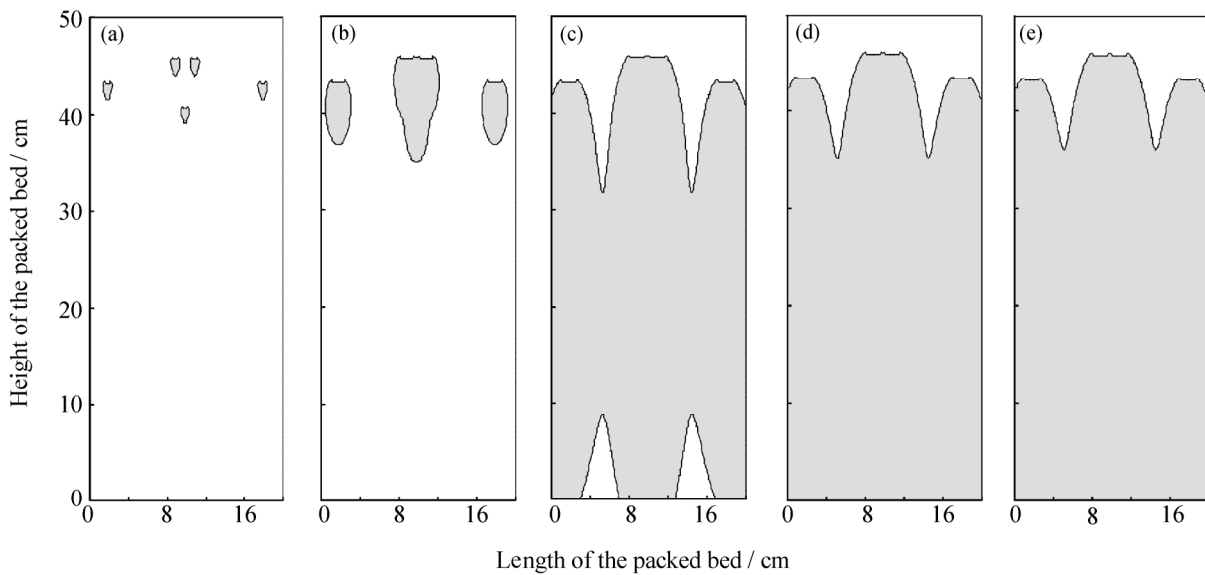
The probability model is also solved within the entire packed bed zone, however, it has not given a non-steady result up to now. Furthermore, if the model-defined dispersal range of a liquid stream is less than  $\pi/4$ , and the flowing liquid in bed receives no other forces besides the action from bed and gravity, this model predicts that the liquid stream flows downwards in a line and the packed bed has no dispersal action on the liquid stream. This does not accord with the facts [11-12]. In the probability model, the mean of the distribution function of the stochastic velocity variable is zero, and as a model parameter, the variable that has positive correlation to the variance of the distribution function of the stochastic velocity variable is adjusted to fit the model-calculated results to the experimental data. The specific distribution function of the stochastic velocity variable in the probability model may be replaced by other distribution functions (for example, the normal distribution

whose mean is zero). As long as the variance of the normal distribution is adjusted, the predicted data can be fitted to the experimental data. Since the distribution function of the stochastic velocity variable may

be selected in a fairly wide extent, the selection is impossible to avoid subjectivity. The probability model may have some limitations in description of the liquid flow inside the coke bed within the blast furnace.



**Fig. 6.** Comparison of the measurement of the liquid flowing region with its prediction by this model, when the inlet total liquid flowrate=25 L/h and the total blast gas flowrate in the horizontal direction is equal to 0 (a), 50 (b), 75 (c), and 100 Nm<sup>3</sup>/h (d), respectively.



**Fig. 7.** Predicted liquid flowing zone inside the bed for 0.05 (a), 0.5 (b), 5 (c), 10 (d), and 60 s (e), respectively, after the five point-liquid-stream sources are introduced synchronously.

Based on the above analysis, it is seen that the present model is considerably more complete in mathematical descriptions compared with other existent models, and will be considerably better in the simulation on the ironmaking process.

## 5. Conclusions

(1) The actions of a packed bed to liquid are considerably more completely described by three assumptions, and a new mathematical model is established to describe the momentum transport phenomena inside the packed bed utilized in the ironmaking process. This model can be used in the whole bed zone packed by all particles, and can also give the solutions of a non-steady state; the scale of numerical grids for the solution of the model should not correspond to the size of packed particles.

(2) The model-predicted results are in fairly good agreements with the experimental data.

(3) The established model can predict the liquid flowing zone and the distribution of liquid flowrate in the packed bed, and will be considerably better applied in the simulation on the ironmaking process compared with other existent models.

## References

- [1] J. Yagi, Mathematical modeling of the flow of four fluids in a packed bed, *ISIJ Int.*, 33(1993), No.6, p.619.
- [2] T. Sugiyama, A. Nakagawa, H. Shibaike, *et al.*, Analysis on liquid flow in the dripping zone of blast furnace, *Tetsu-to-Hagané* (in Japanese), 73(1987), No.15, p.242.
- [3] P.R. Austin, H. Nogami, and J. Yagi, Computational investigation of scrap charging to the blast furnace, *ISIJ Int.*, 38(1998), No.7, p.697.
- [4] J.A. Castro, H. Nogami, and J. Yagi, Numerical investigation of simultaneous injection of pulverized coal and natural gas with oxygen enrichment to the blast furnace, *ISIJ Int.*, 42(2002), No.11, p.1203.
- [5] M. Chu, H. Nogami, and J. Yagi, Numerical analysis on blast furnace performance under operation with top gas recycling and carbon composite agglomerates charging, *ISIJ Int.*, 44(2004), No.12, p.2159.
- [6] M. Chu, X. Guo, F. Shen, *et al.*, Multi-fluid mathematical model for blast furnace and its solution, *J. Northeast. Univ. Nat. Sci.* (in Chinese), 28(2007), No.3, p.361.
- [7] Y. Ohno and M. Schneider, Effect of horizontal gas flow on liquid dropping flow in two-dimensional packed bed, *Tetsu-to-Hagané* (in Japanese), 74(1988), No.10, p.35.
- [8] M. Matsu-ura and Y. Ohno, Modeling of gas and liquid flow in two dimensional packed bed and analysis of dropping zone in blast furnace, *Tetsu-to-Hagané* (in Japanese), 80(1994), No.12, p.14.
- [9] J. Wang, R. Kakahashi, and J. Yagi, Simulation model of the gas-liquid flows in the packed bed, *Tetsu-to-Hagané* (in Japanese), 77(1991), No.10, p.47.
- [10] Y. Eto, K. Takeda, S. Miyagawa, *et al.*, Experiments and simulation of the liquid flow in the dropping zone of a blast furnace, *ISIJ Int.*, 33(1993), No.6, p.681.
- [11] G.X. Wang, S.J. Chew, A.B. Yu, *et al.*, Modeling the discontinuous liquid flow in a blast furnace, *Metall. Mater. Trans. B*, 28(1997), No.2, p.333.
- [12] C.S. Wang, *Numerical Simulation for Final Reduction Reactor of a Two-Step Method Capable of Producing Hot Metal Containing Cr* [Dissertation] (in Chinese), Shanghai University, 2005, p.38.
- [13] G.S. Gupta, J.D. Litster, V.R. Rudolph, *et al.*, Model studies of liquid flow in the blast furnace lower zone, *ISIJ Int.*, 36(1996), No.1, p.32.
- [14] G.S. Gupta, J.D. Litster, E.T. White, *et al.*, Nonwetting flow of a liquid through a packed bed with gas cross-flow, *Metall. Mater. Trans. B*, 28(1997), No.4, p.597.
- [15] S.J. Chew, P. Zulli, and A.B. Yu, Modeling of liquid flow in the blast furnace, *ISIJ Int.*, 41(2001), No.10, p.1112.
- [16] G.X. Wang, D.Y. Liu, J.D. Litster, *et al.*, Experimental and numerical simulation of discrete liquid flow in a packed bed, *Chem. Eng. Sci.*, 52(1997), No.21-22, p.4013.
- [17] D.Y. Liu, G.X. Wang, and J.D. Litster, Insaturated liquid percolation flow through nonwetted packed beds, *AIChE J.*, 48(2002), No.5, p.953.
- [18] B.Y. Xu and X.S. Liu, *Applied Elastic-Plastic Mechanics* (in Chinese), Tsinghua University Press, Beijing, 1995, p.7.
- [19] D.Y. Liu, G.X. Wang, and J.D. Litster, Experimental investigation of liquid flow shift due to gas cross flow in non-wetted packed beds, *ISIJ Int.*, 41(2001), No.1, p.10.